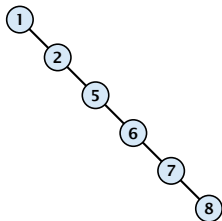
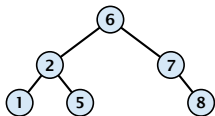


## 7.1 Binary Search Trees

An (**internal**) **binary search tree** stores the elements in a binary tree. Each tree-node corresponds to an element. All elements in the left sub-tree of a node  $v$  have a smaller key-value than  $\text{key}[v]$  and elements in the right sub-tree have a larger-key value. We assume that all key-values are different.

(**External** Search Trees store objects only at leaf-vertices)

Examples:

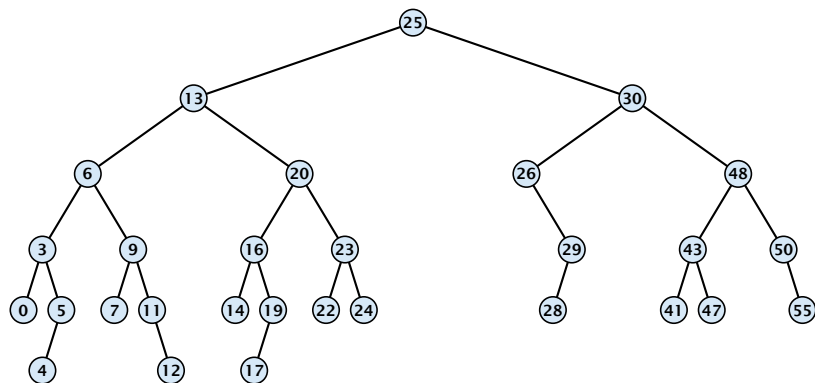


## 7.1 Binary Search Trees

We consider the following operations on binary search trees. Note that this is a super-set of the dictionary-operations.

- ▶  $T.\text{insert}(x)$
- ▶  $T.\text{delete}(x)$
- ▶  $T.\text{search}(k)$
- ▶  $T.\text{successor}(x)$
- ▶  $T.\text{predecessor}(x)$
- ▶  $T.\text{minimum}()$
- ▶  $T.\text{maximum}()$

# Binary Search Trees: Searching

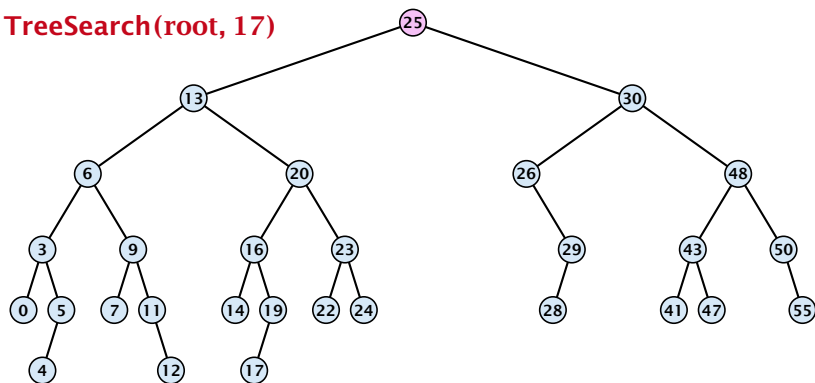


## Algorithm 1 $\text{TreeSearch}(x, k)$

- 1: **if**  $x = \text{null}$  **or**  $k = \text{key}[x]$  **return**  $x$
- 2: **if**  $k < \text{key}[x]$  **return**  $\text{TreeSearch}(\text{left}[x], k)$
- 3: **else return**  $\text{TreeSearch}(\text{right}[x], k)$

# Binary Search Trees: Searching

TreeSearch(root, 17)

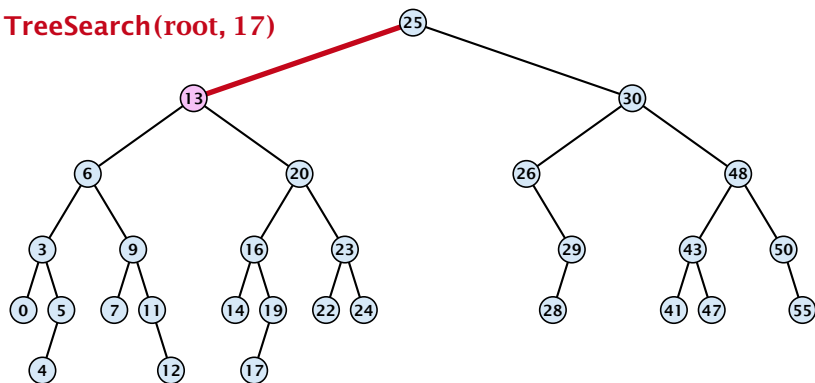


## Algorithm 1 TreeSearch( $x, k$ )

- 1: **if**  $x = \text{null}$  **or**  $k = \text{key}[x]$  **return**  $x$
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# Binary Search Trees: Searching

TreeSearch(root, 17)

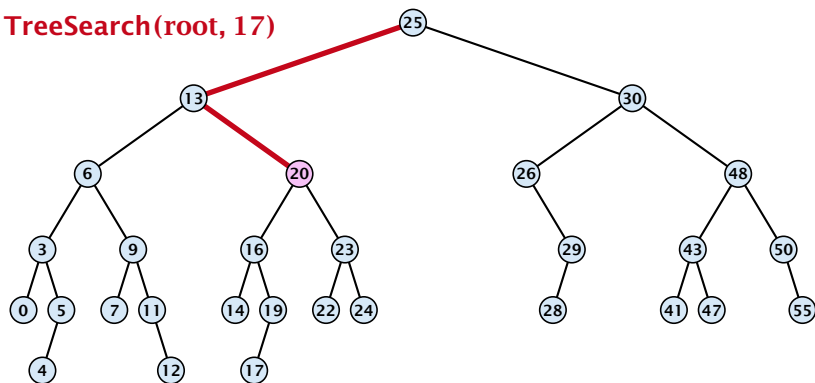


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# Binary Search Trees: Searching

TreeSearch(root, 17)

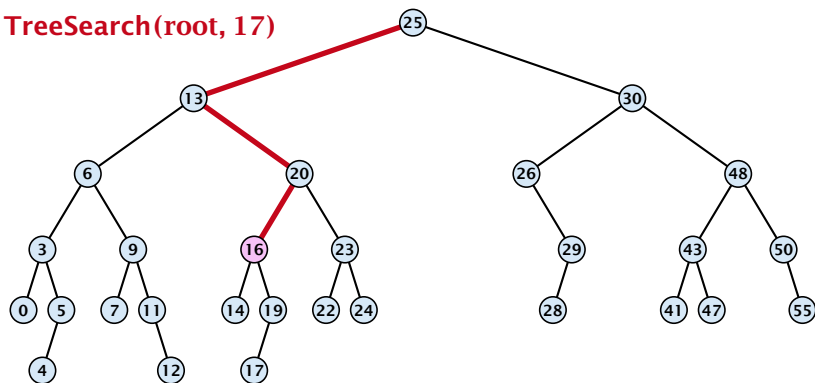


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# Binary Search Trees: Searching

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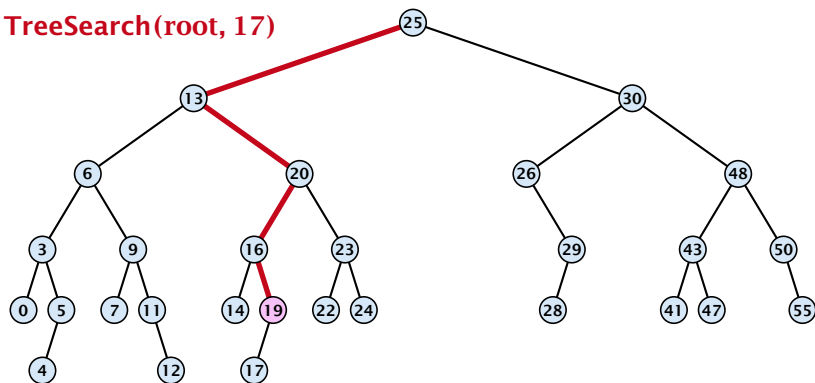


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- 1: **if**  $x = \text{null}$  **or**  $k = \text{key}[x]$  **return**  $x$
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# Binary Search Trees: Searching

TreeSearch(root, 17)



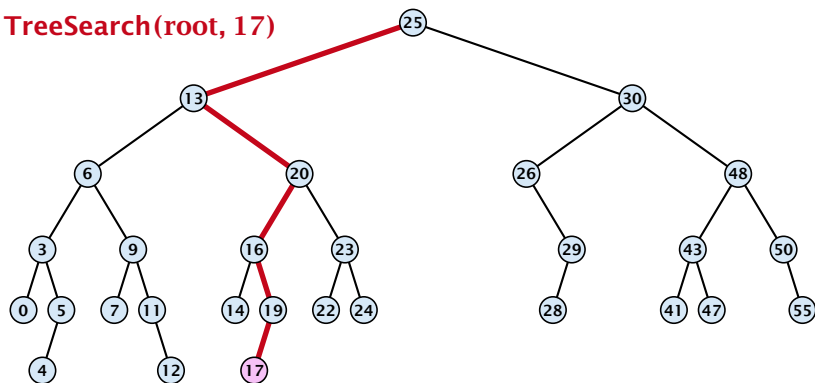
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# Binary Search Trees: Searching

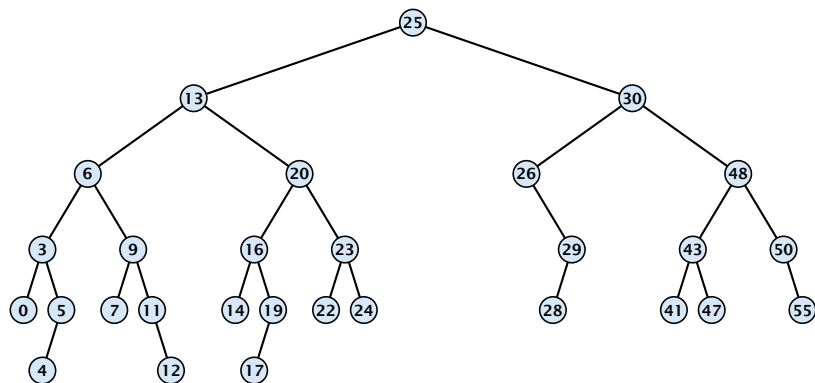
TreeSearch(root, 17)



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# Binary Search Trees: Searching

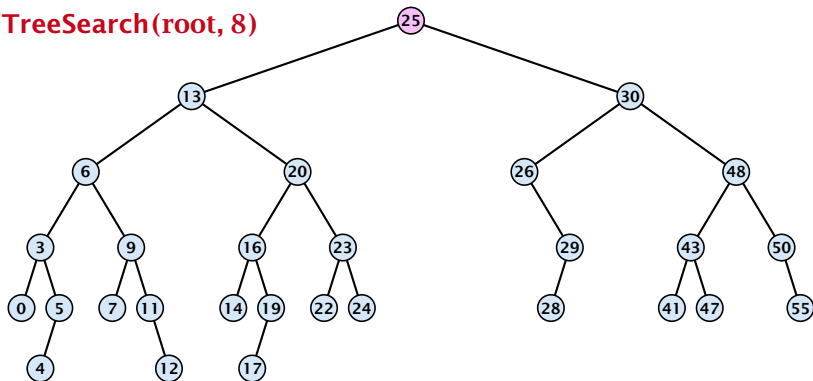


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# Binary Search Trees: Searching

TreeSearch(root, 8)

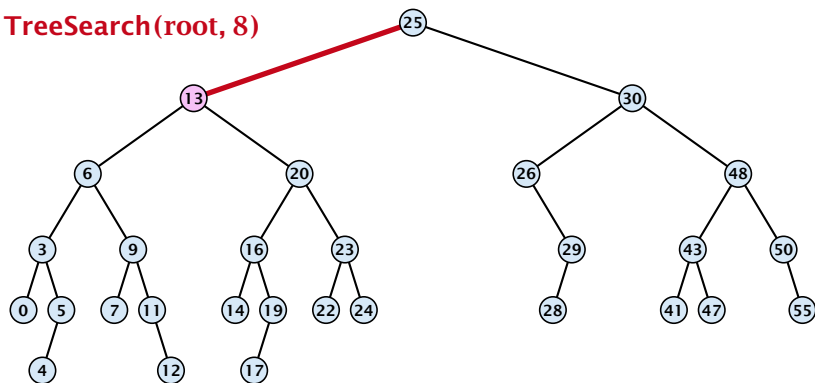


## Algorithm 1 TreeSearch( $x, k$ )

- 1: **if**  $x = \text{null}$  **or**  $k = \text{key}[x]$  **return**  $x$
- 2: **if**  $k < \text{key}[x]$  **return** TreeSearch(left[ $x$ ],  $k$ )
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TreeSearch(root, 8)

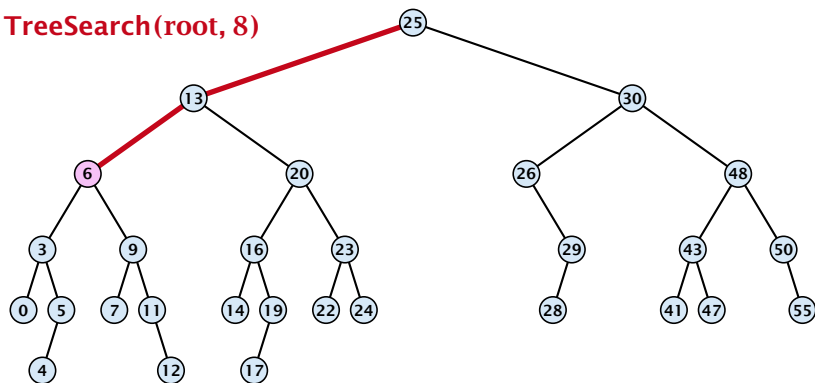


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TreeSearch(root, 8)

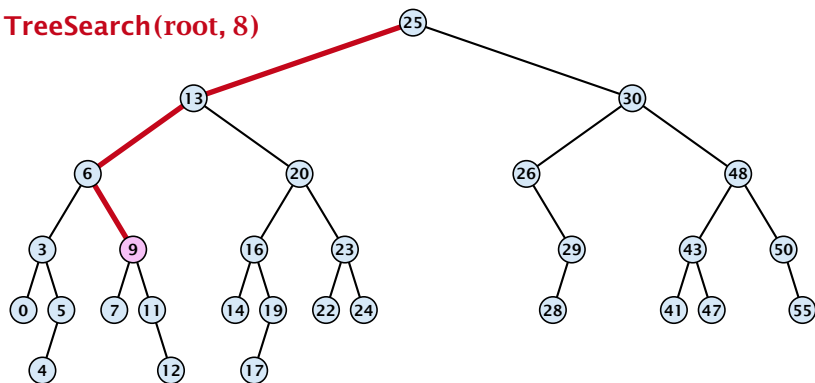


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TreeSearch(root, 8)

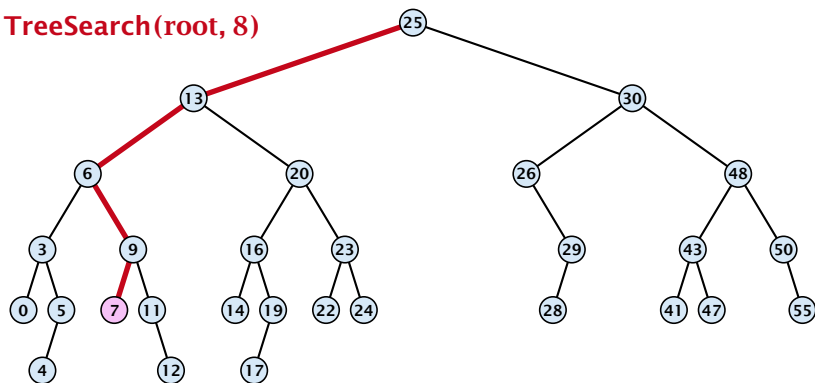


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TreeSearch(root, 8)

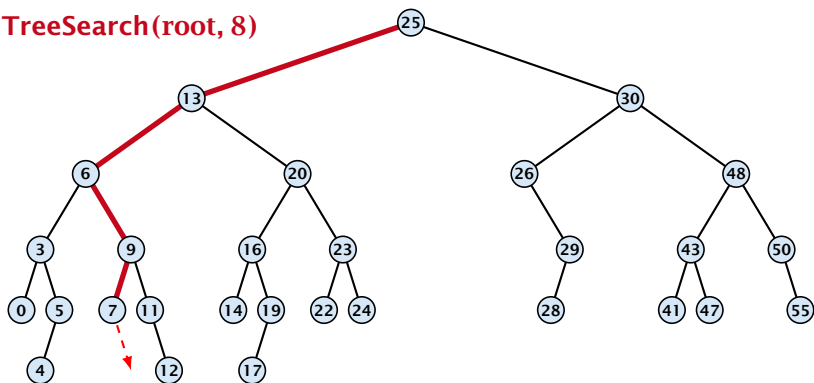


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TreeSearch(root, 8)

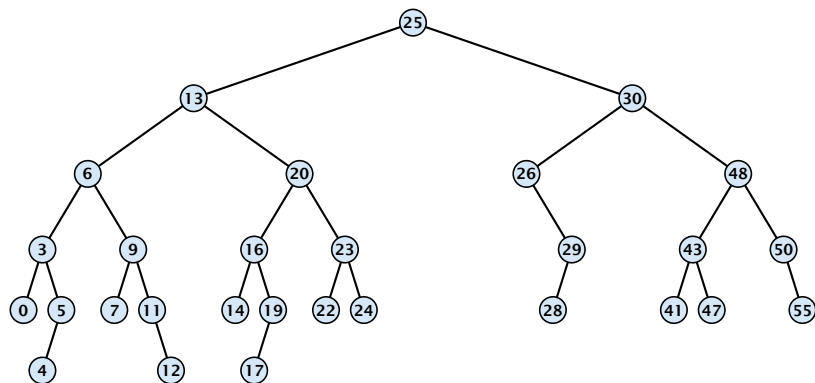


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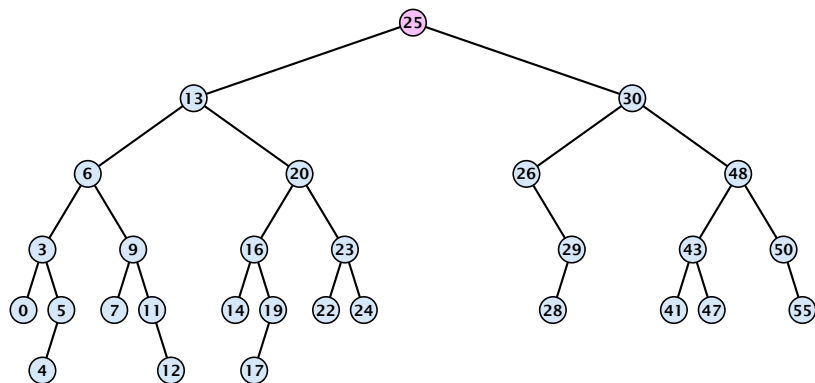
# Binary Search Trees: Minimum



## Algorithm 2 TreeMin( $x$ )

- 1: **if**  $x = \text{null}$  **or**  $\text{left}[x] = \text{null}$  **return**  $x$
- 2: **return**  $\text{TreeMin}(\text{left}[x])$

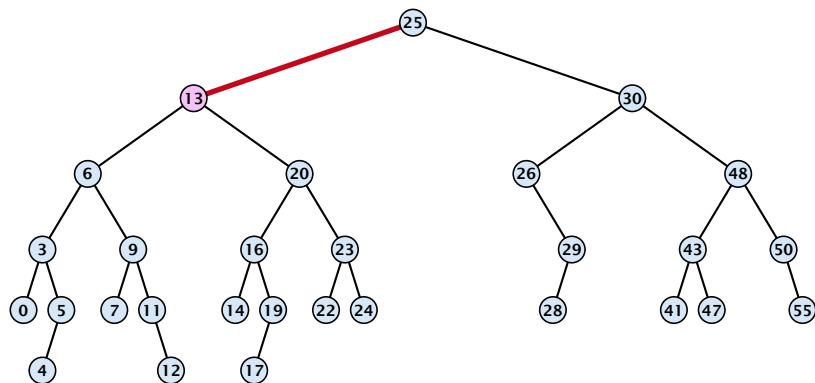
# Binary Search Trees: Minimum



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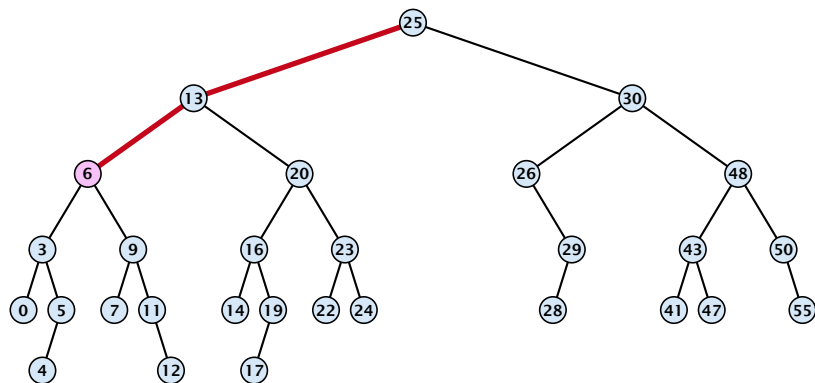
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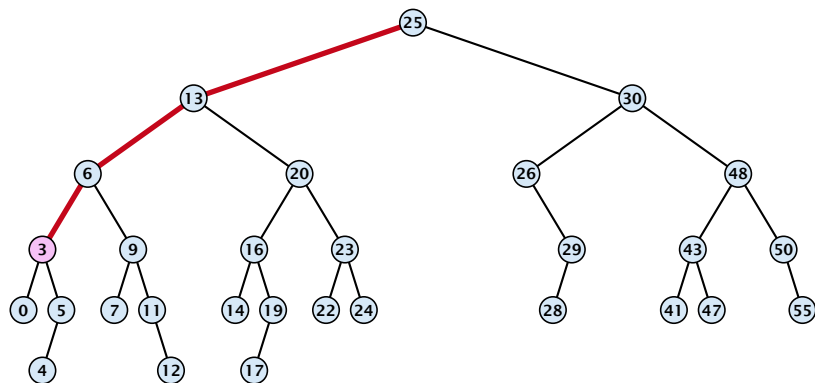
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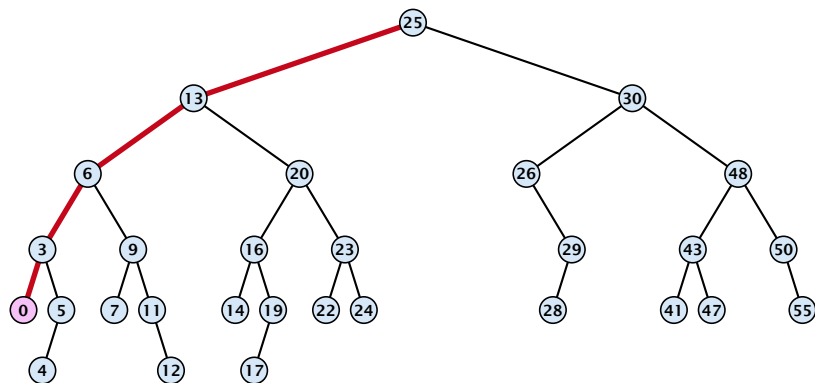
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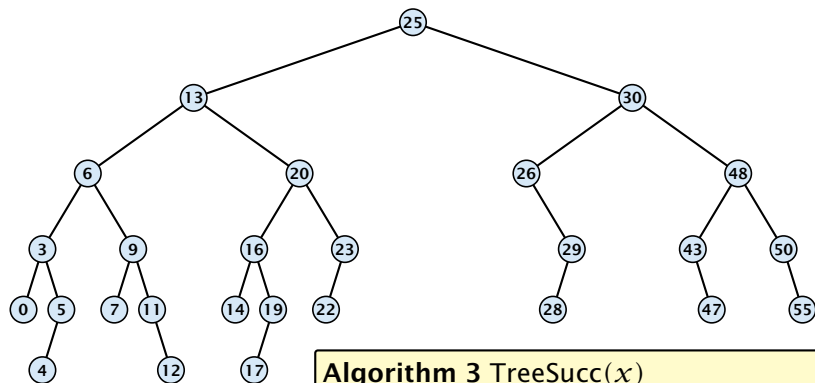
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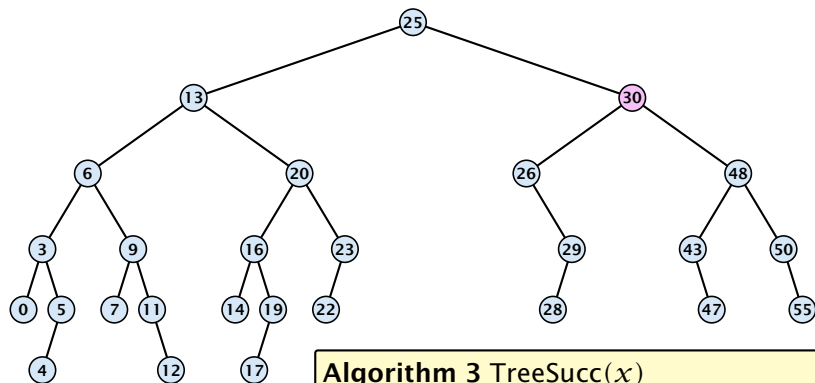
# Binary Search Trees: Successor



## Algorithm 3 TreeSucc( $x$ )

- 1: **if**  $\text{right}[x] \neq \text{null}$  **return**  $\text{TreeMin}(\text{right}[x])$
- 2:  $y \leftarrow \text{parent}[x]$
- 3: **while**  $y \neq \text{null}$  **and**  $x = \text{right}[y]$  **do**
- 4:      $x \leftarrow y; y \leftarrow \text{parent}[x]$
- 5: **return**  $y$ ;

# Binary Search Trees: Successor

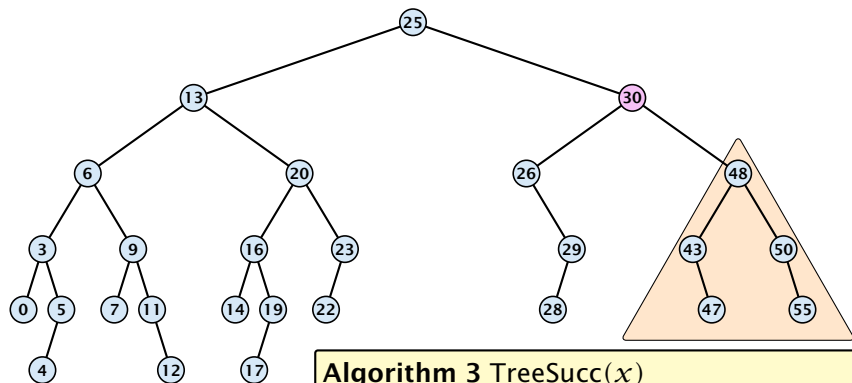


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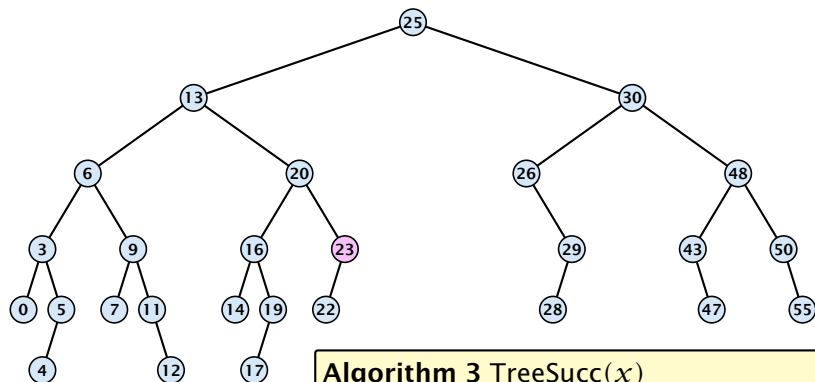
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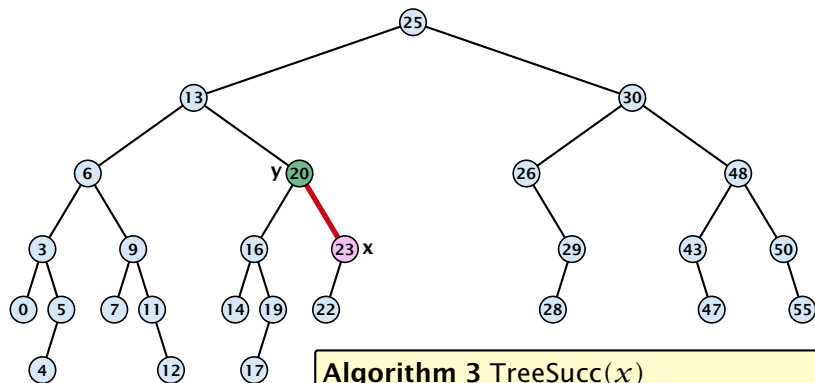
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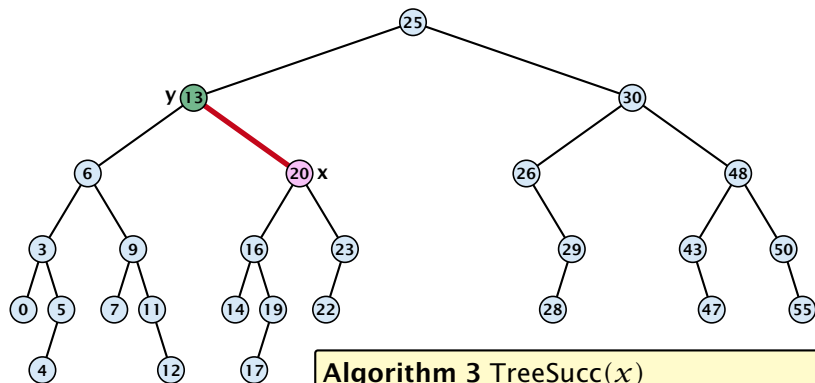
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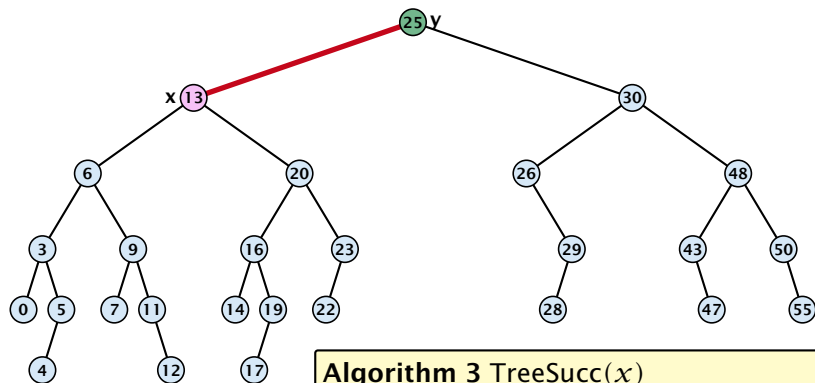
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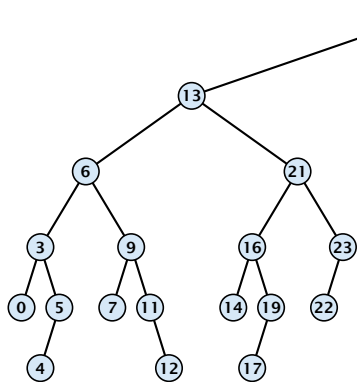
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## Binary Search Trees: Insert

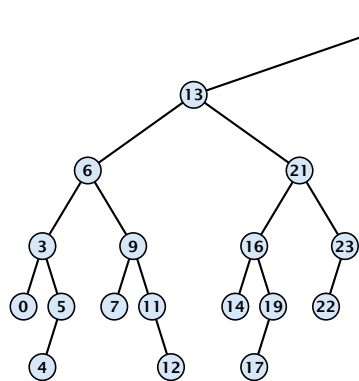


### Algorithm 4 TreeInsert( $x, z$ )

```
1: if  $x = \text{null}$  then  
2:    $\text{root}[T] \leftarrow z$ ;  $\text{parent}[z] \leftarrow \text{null}$ ;  
3:   return;  
4: if  $\text{key}[x] > \text{key}[z]$  then  
5:   if  $\text{left}[x] = \text{null}$  then  
6:      $\text{left}[x] \leftarrow z$ ;  $\text{parent}[z] \leftarrow x$ ;  
7:   else TreeInsert( $\text{left}[x], z$ );  
8: else  
9:   if  $\text{right}[x] = \text{null}$  then  
10:     $\text{right}[x] \leftarrow z$ ;  $\text{parent}[z] \leftarrow x$ ;  
11:   else TreeInsert( $\text{right}[x], z$ );
```

# Binary Search Trees: Insert

Insert element **not** in the tree.

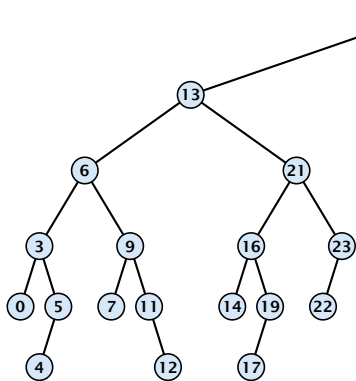


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- 2:      $\text{root}[T] \leftarrow z$ ;  $\text{parent}[z] \leftarrow \text{null}$ ;
- 3:     **return**;
- 4: **if**  $\text{key}[x] > \text{key}[z]$  **then**
- 5:     **if**  $\text{left}[x] = \text{null}$  **then**
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# Binary Search Trees: Insert

Insert element **not** in the tree.



Search for  $z$ . At some point the search stops at a null-pointer. This is the place to insert  $z$ .

## Algorithm 4 TreeInsert( $x, z$ )

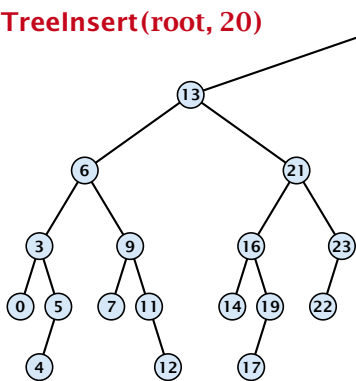
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3:   return;
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# Binary Search Trees: Insert

Insert element **not** in the tree.

**TreeInsert**(root, 20)



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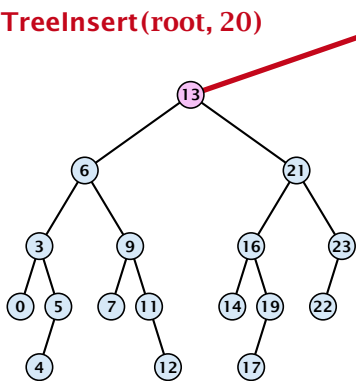
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3:   return;
4: if key[ $x$ ] > key[ $z$ ] then
5:   if left[ $x$ ] = null then
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**TreeInsert**(root, 20)



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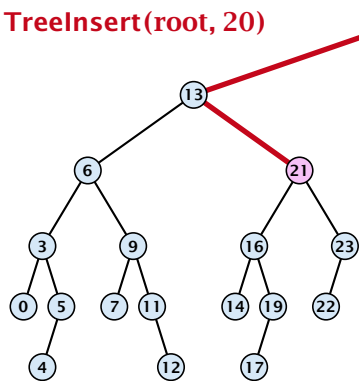
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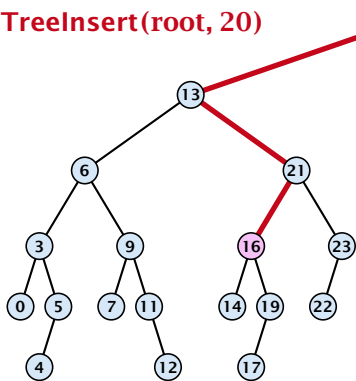
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Insert element **not** in the tree.

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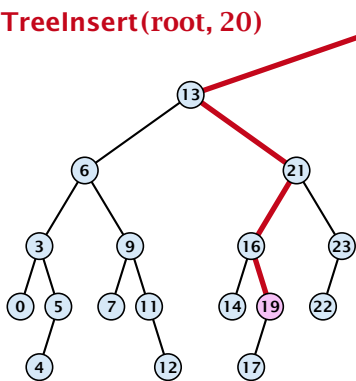
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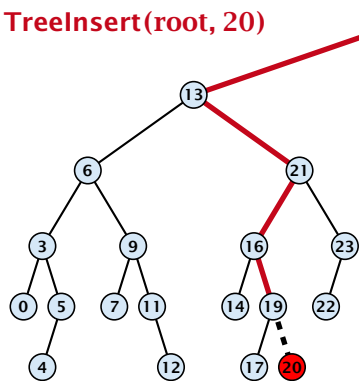
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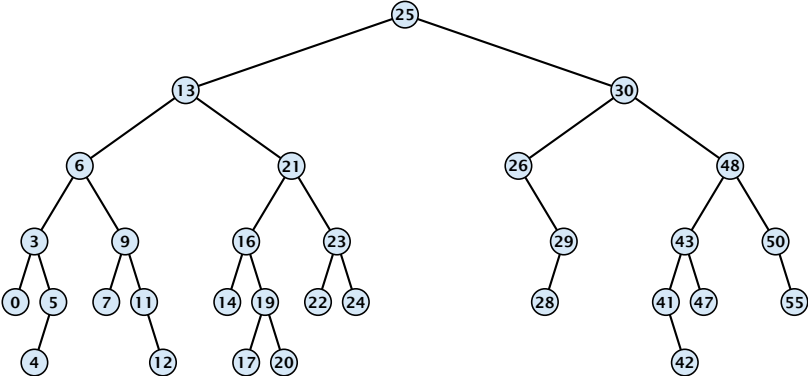


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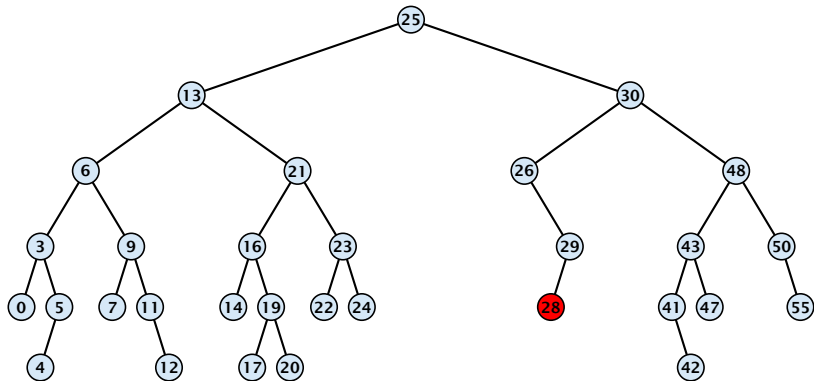
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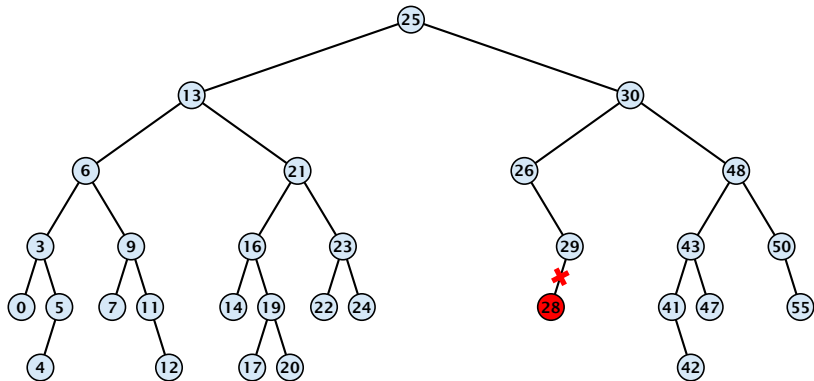
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Element does not have any children

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# Binary Search Trees: Delete

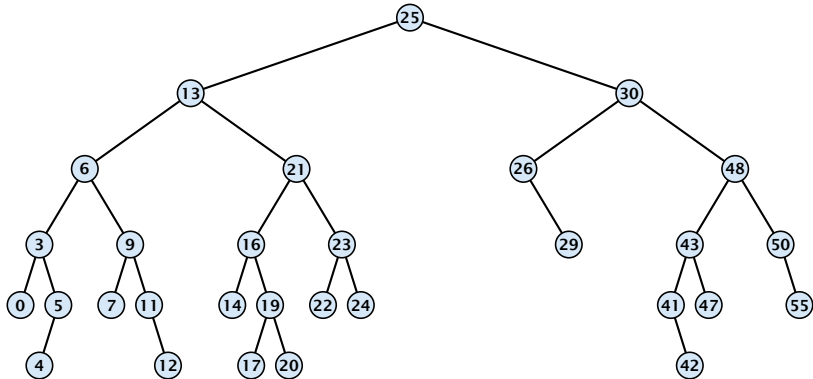


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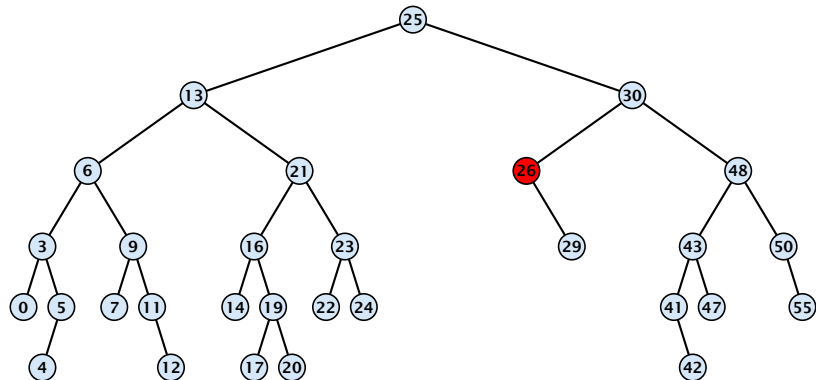


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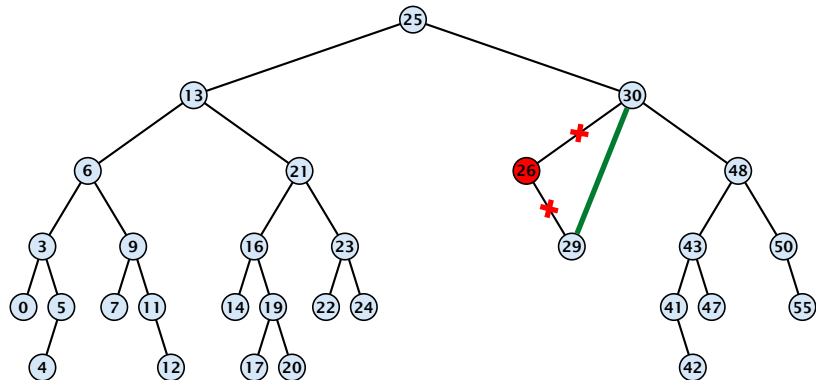


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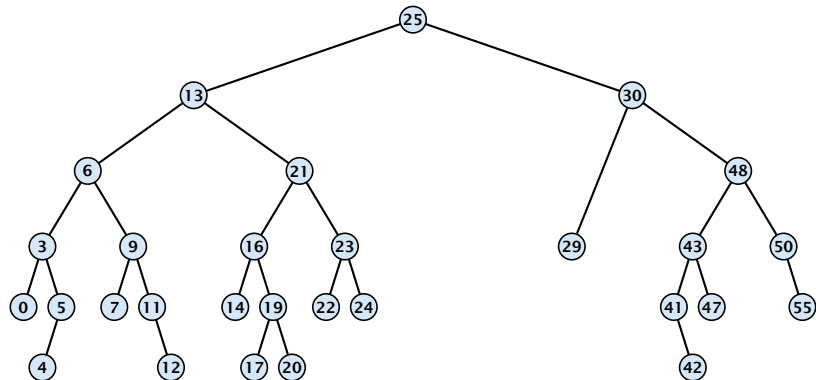


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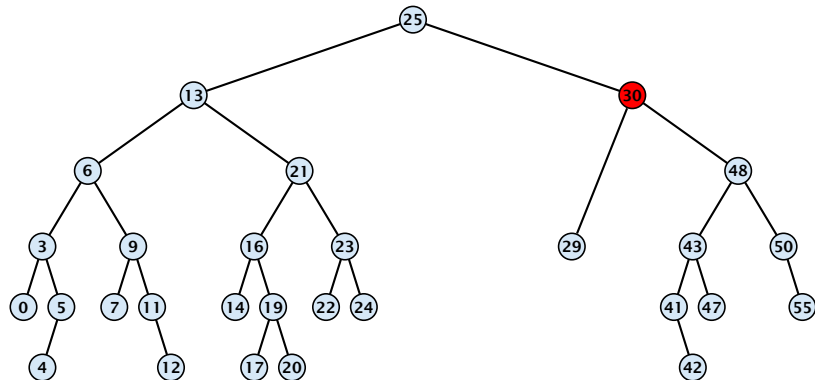


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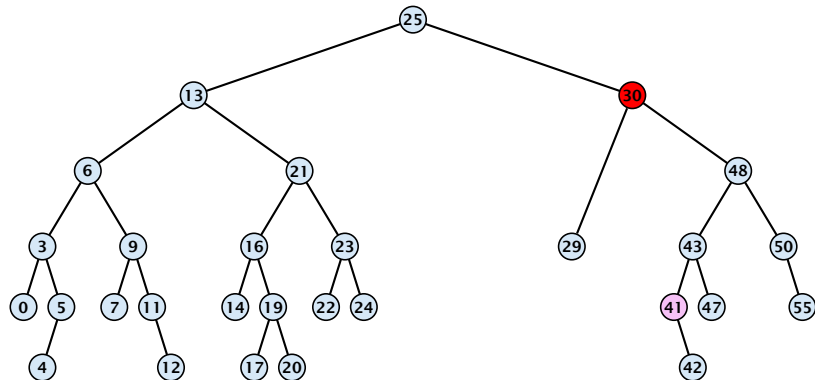


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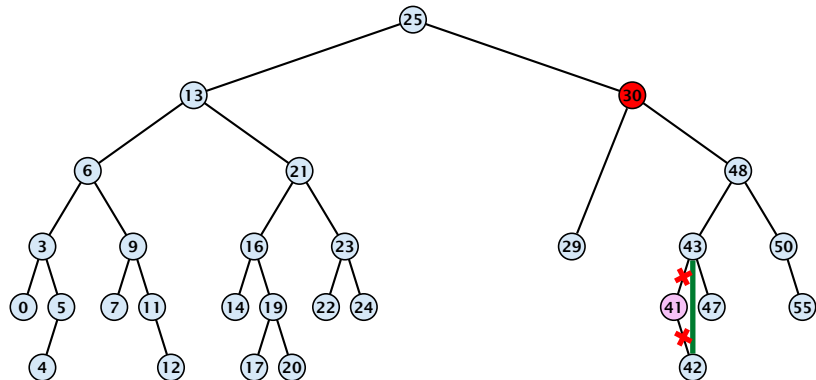


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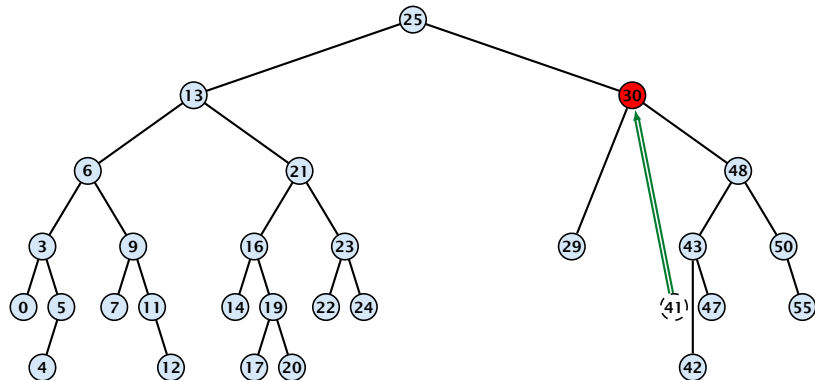
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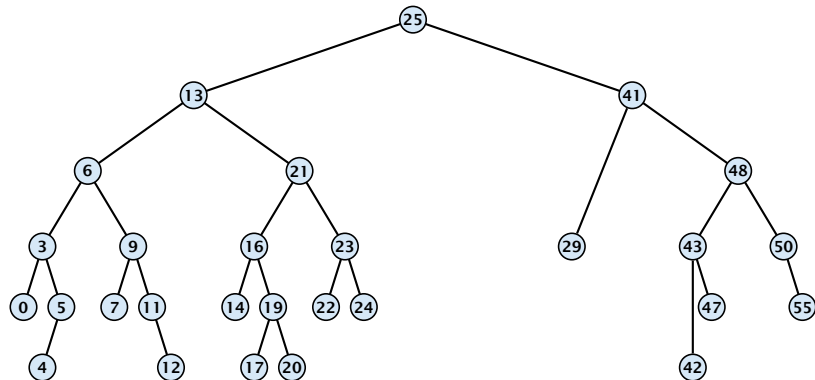


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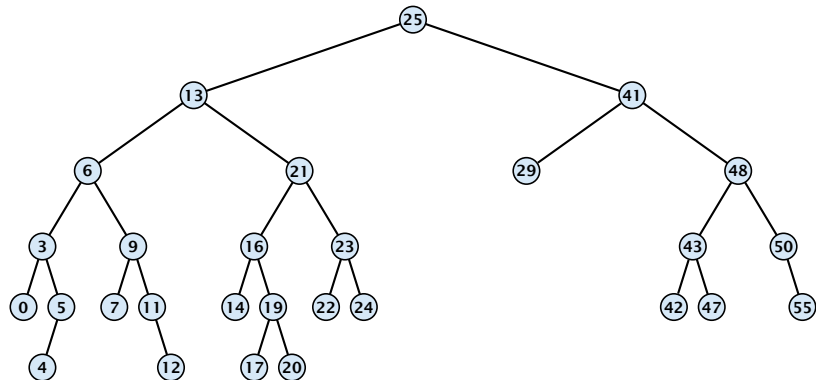


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# Binary Search Trees: Delete

## Algorithm 9 TreeDelete( $z$ )

```
1: if left[ $z$ ] = null or right[ $z$ ] = null
2:   then  $y \leftarrow z$  else  $y \leftarrow \text{TreeSucc}(z)$ ;   select  $y$  to splice out
3:   if left[ $y$ ]  $\neq$  null
4:     then  $x \leftarrow \text{left}[y]$  else  $x \leftarrow \text{right}[y]$ ;  $x$  is child of  $y$  (or null)
5:   if  $x \neq \text{null}$  then parent[ $x$ ]  $\leftarrow$  parent[ $y$ ];   parent[ $x$ ] is correct
6:   if parent[ $y$ ] = null then
7:     root[ $T$ ]  $\leftarrow x$ 
8:   else
9:     if  $y = \text{left}[\text{parent}[y]]$  then
10:      left[parent[ $y$ ]]  $\leftarrow x$ 
11:     else
12:      right[parent[ $y$ ]]  $\leftarrow x$ 
13:   if  $y \neq z$  then copy  $y$ -data to  $z$ 
```

} fix pointer to  $x$

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AVL-trees, Red-black trees, Scapegoat trees, 2-3 trees, B-trees, AA trees, Treaps

similar: SPLAY trees.