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$\mathcal{S}$ is the set of subsets that separate $s$ from $t$.

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The Separation Problem for the Shortest Path LP is the Minimum Cut Problem.

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# 18 Cuts & Metrics

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## How do we round the LP?

- Let $B(s,r)$ be the ball of radius $r$ around $s$ (w.r.t. metric $d$). Formally:

$$B = \{v \in V \mid d(s,v) \le r\}$$

- For $0 \le r < 1$, $B(s,r)$ is an $s$-$t$-cut.

**Which value of $r$ should we choose? choose randomly!!!**

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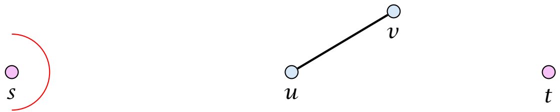

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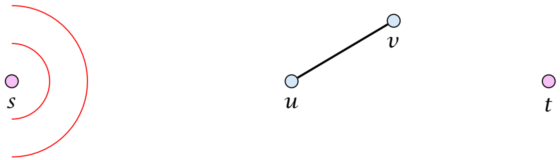

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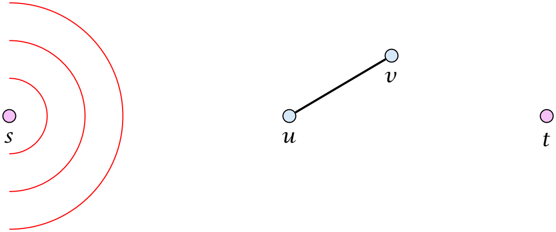

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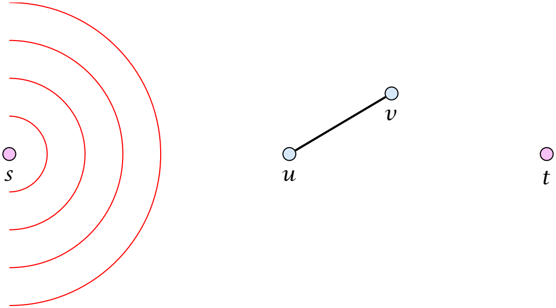

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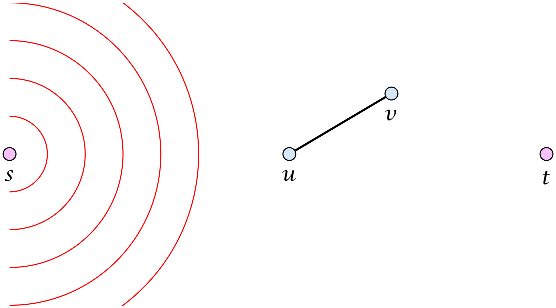

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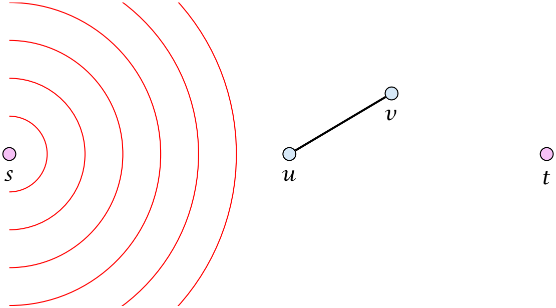

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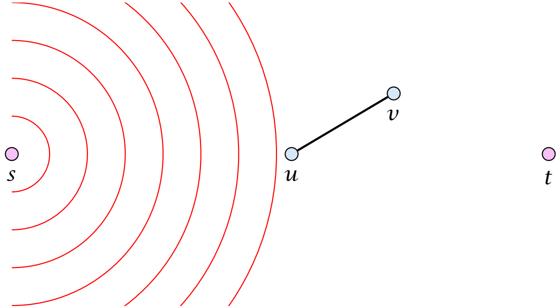

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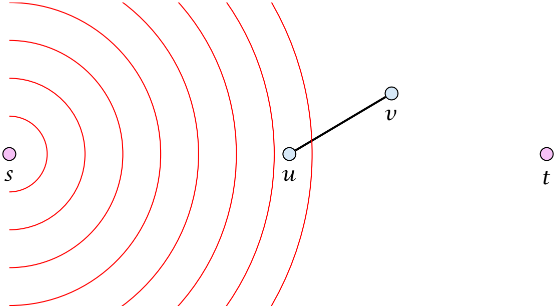

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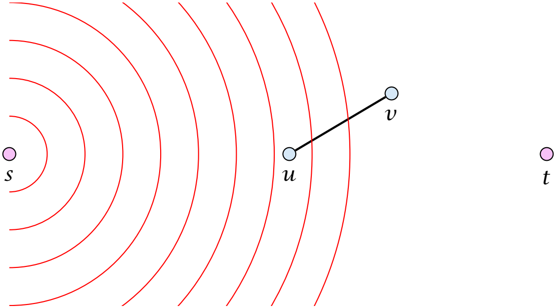

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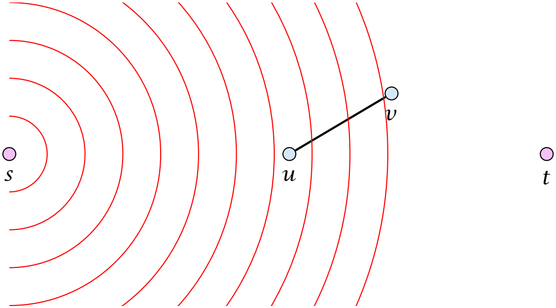

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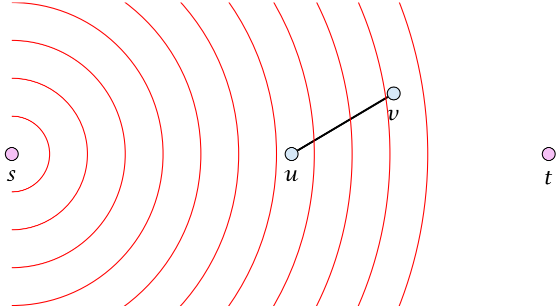

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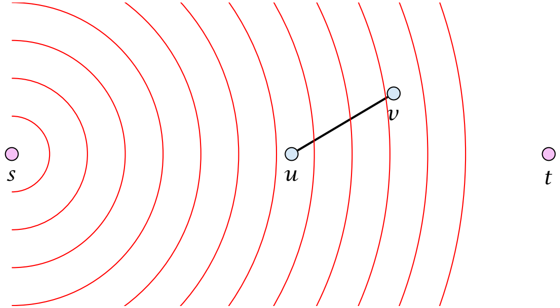

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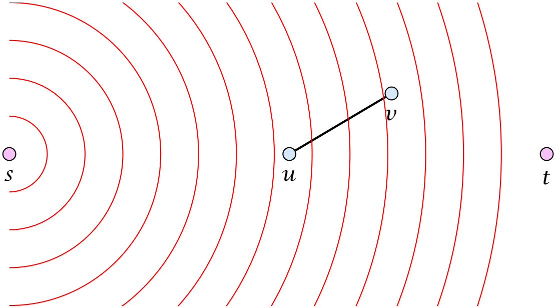

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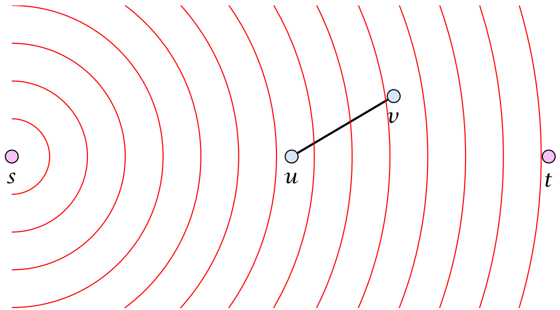

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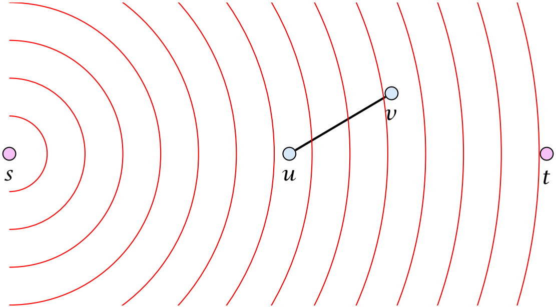


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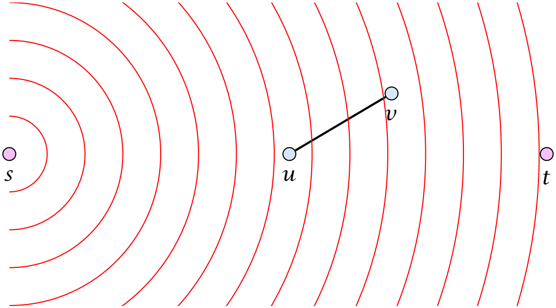


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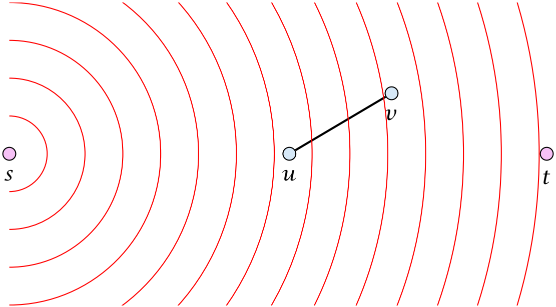


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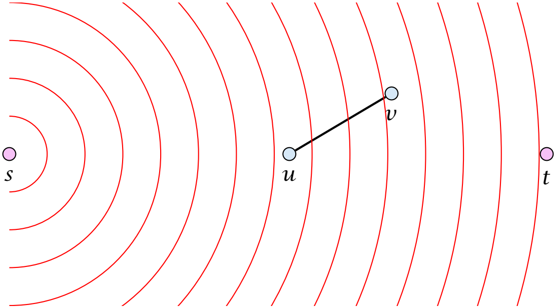


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On the other hand:

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Given a graph $G = (V, E)$, together with source-target pairs $s_i, t_i$, $i = 1, \dots, k$, and a capacity function $c : E \to \mathbb{R}^+$ on the edges. Find a subset $F \subseteq E$ of the edges such that all $s_i$-$t_i$ pairs lie in different components in $G = (V, E \setminus F)$.

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- Assume for simplicity that all edge-length $\ell_e$ are multiples of $\delta \ll 1$.
- Replace the graph $G$ by a graph $G'$, where an edge of length $\ell_e$ is replaced by $\ell_e/\delta$ edges of length $\delta$.
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z ← 0
repeat
    flip a coin (Pr[heads] = p)
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- we make $\frac{1}{2\delta}$ trials before reaching radius $1/2$.
- we say a Region Growing is not successful if it does not terminate before reaching radius $1/2$.

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If we are not successful we simply perform a trivial $k$-approximation.

This only increases the expected cost by at most $\frac{1}{k^2} \cdot k\text{OPT} \leq \text{OPT}/k$.

Hence, our final cost is $\mathcal{O}(\ln k) \cdot \text{OPT}$ in expectation.