Technische Universität München
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Winter Semester 2017/18
Problem Set 12
January 29, 2018

Online and Approximation Algorithms

Due February 5, 2018 at 10:00

Exercise 1 (Max-NAE 3-SAT – 10 points)

We consider the *Max-NAE 3-SAT* (Max Not-All-Equal 3-SAT) problem. In this problem we are given a *3CNF* formula and our goal is to find a variable assignment, such that the number of clauses containing at least one true **and** one false literal is maximized.
Develop a randomized approximation algorithm for *Max-NAE 3-SAT* with an approximation ratio of $\frac{3}{4}$.

Exercise 2 (Set Cover and Vertex Cover – 10 points)

Recall that in the Vertex Cover problem, one has to select a subset of vertices from a graph G such that every edge in G is incident to at least one vertex of this set. The goal is to find such a set of minimum cost.
Let 2SC be the Set Cover problem restricted to instances where each item appears in at most two sets.
Show that 2SC is equivalent to the Vertex Cover problem with arbitrary costs. That is, show that an instance of the Vertex Cover problem can be reduced to an instance of 2SC and that for both problems the same approximation ratio can be obtained.

Exercise 3 (Scheduling with Arrival Times – 10 points)

Consider the following variant of the makespan minimization problem: Schedule n jobs $J_1, \ldots, J_n$ with processing times $p_1, \ldots, p_n$ and arrival times $a_1, \ldots, a_n$ to m identical parallel machines such that the makespan is minimized, i.e. the completion time of the last job that finishes in the schedule. A job cannot be processed before its arrival time. All processing and arrival times are known from the beginning.
Propose a 2-approximative algorithm.
Hint: For your analysis differentiate between the phases where all machines are running and those where at least one machine is idle, but do not finished its jobs.

Exercise 4 (ILP and duality – 10 points)

Let $G = (V, E)$ be a connected, undirected graph. Consider the following problem:

VertexCover: Find a subset $U \subseteq V$ with minimal size such that each edge $\{u, v\} \in E$ is covered by U, i.e. $\{u, v\} \cap U \neq \emptyset$.

(a) Give an ILP formulation for VertexCover.

(b) Determine the dual program and interpret the corresponding problem.