Technische Universität München
Fakultät für Informatik
Lehrstuhl für Theoretische Informatik
Prof. Dr. Susanne Albers
Jens Quedenfeld

Winter Semester 2017/18
Problem Set 9
January 8, 2018

Online and Approximation Algorithms

Due January 15, 2018 at 10:00

Exercise 1 (Online search problem – 10 points)

Recall the online search problem presented in class. Assume that only $\varphi = \frac{M}{m}$ is known, i.e. m and M are unknown. Prove that there is no deterministic algorithm that achieves a competitive ratio that is smaller than φ.

Exercise 2 (EXPO – 10 points)

Recall the online search problem presented in class and the EXPO algorithm for solving it. Moreover, let μ be a probability distribution of the natural numbers $\mathbb{N} = \{0, 1, 2, \dots\}$ according to which the number i is chosen with probability q_i. Now, consider the EXPO(μ) algorithm which is defined as follows: Choose the price $p \cdot 2^i$ with probability q_i, where p is the first price revealed.
Show that EXPO(μ) is $\frac{2}{q_j}$-competitive, for some $j \in \mathbb{N}$.

Exercise 3 (k-Server on a Line – 10 points)

Consider the k-server problem where all servers and requests are located on a continuous straight line. At the beginning all servers have the same position. Algorithm DC (*Double Coverage*) serves each incoming request for point x as follows.
If x is on the left of all servers, move the closest server to it. Treat the case where x is on the right of all servers similarly. Otherwise x is located between two servers s_i and s_j. Move both servers with equal speed towards x until one of them reaches x (i.e., if s_i is the closest, then both servers move distance $d(s_i, x)$).
Let $s_1, s_2, \dots, s_k$ and $a_1, a_2, \dots, a_k$ be the locations of the servers by DC and OPT, respectively. We define the potential function $\Phi = k \cdot M + D$, where M is the minimum cost perfect matching in the bipartite graph between $s_1, s_2, \dots, s_k$ and $a_1, a_2, \dots, a_k$, while $D = \sum_{i<j} d(s_i, s_j)$ is the sum of all pairwise distances between the servers of DC.

(a) Show that Φ satisfies the following properties:

  (i) If the adversary's cost increases by y, then the change in the potential is $\Delta\Phi \leq ky$.

  (ii) If the cost of DC increases by y', then the change in the potential is $\Delta\Phi \leq -y'$.

(b) Show that DC is k-competitive.

Exercise 4 (Max Cut – 10 points)

In the lecture, a deterministic $\frac{1}{2}$-approximation algorithm for the Max Cut problem was given.

Consider the following randomized algorithm to solve Max Cut. Each vertex is randomly and independently assigned a value 0 or 1. All vertices with value 1 are in S and all vertices with value 0 are in $V \setminus S$. Prove that the approximation ratio of this algorithm is also $\frac{1}{2}$.