5.2 Simplex and Duality

The following linear programs form a primal dual pair:

$$z = \max\{c^T x \mid Ax = b, x \ge 0\}$$
$$w = \min\{b^T y \mid A^T y \ge c\}$$

This means for computing the dual of a standard form LP, we do not have non-negativity constraints for the dual variables.

$\max\{c^T x \mid Ax = b, x \ge 0\}$

5.2 Simplex and Duality

The following linear programs form a primal dual pair: $z = \max\{c^T x \mid Ax = b, x \ge 0\}$ $w = \min\{b^T \gamma \mid A^T \gamma \ge c\}$

This means for computing the dual of a standard form LP, we do not have non-negativity constraints for the dual variables.

5.2 Simplex and Duality

- 5.2 Simplex and Duality

Primal:

Proof

$$\max\{c^T x \mid Ax = b, x \ge 0\}$$

$$= \max\{c^T x \mid Ax \le b, -Ax \le -b, x \ge 0\}$$

The following linear programs form a primal dual pair:

5.2 Simplex and Duality

$$z = \max\{c^T x \mid Ax = b, x \ge 0\}$$
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This means for computing the dual of a standard form LP, we do

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5.2 Simplex and Duality

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5.2 Simplex and Duality

Primal:

Proof

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$$\max\{c^T x \mid Ax = b, x \ge 0\}$$

$$= \max\{c^T x \mid Ax \le b, -Ax \le -b, x \ge 0\}$$

$$= \max\{c^T x \mid \begin{bmatrix} A \\ -A \end{bmatrix} x \le \begin{bmatrix} b \\ -b \end{bmatrix}, x \ge 0\}$$

5.2 Simplex and Duality

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5.2 Simplex and Duality

not have non-negativity constraints for the dual variables.

Primal:

Proof

$$\max\{c^T x \mid Ax = b, x \ge 0\}$$

$$= \max\{c^T x \mid Ax \le b, -Ax \le -b, x \ge 0\}$$

$$= \max\{c^T x \mid \begin{bmatrix} A \\ -A \end{bmatrix} x \le \begin{bmatrix} b \\ -b \end{bmatrix}, x \ge 0\}$$

$\min\{[b^T - b^T]\gamma \mid [A^T - A^T]\gamma \ge c, \gamma \ge 0\}$

The following linear programs form a primal dual pair:

5.2 Simplex and Duality

- $z = \max\{c^T x \mid Ax = h, x > 0\}$ $w = \min\{b^T v \mid A^T v \ge c\}$
- This means for computing the dual of a standard form LP, we do not have non-negativity constraints for the dual variables.
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$$\max\{c^T x \mid Ax = b, x \ge 0\}$$

$$= \max\{c^T x \mid Ax \le b, -Ax \le -b, x \ge 0\}$$

$$= \max\{c^T x \mid \begin{bmatrix} A \\ -A \end{bmatrix} x \le \begin{bmatrix} b \\ -b \end{bmatrix}, x \ge 0\}$$

Dual:
$$\min\{[b^T - b^T]y \mid [A^T - A^T]y \ge c, y \ge 0\}$$

$$y \mid [A^T - A^T]y \ge c, y \ge 0$$

$$= \min \left\{ \begin{bmatrix} b^T - b^T \end{bmatrix} \cdot \begin{bmatrix} y^+ \\ y^- \end{bmatrix} \middle| \begin{bmatrix} A^T - A^T \end{bmatrix} \cdot \begin{bmatrix} y^+ \\ y^- \end{bmatrix} \ge c, y^- \ge 0, y^+ \ge 0 \right\}$$

5.2 Simplex and Duality

The following linear programs form a primal dual pair:

$$z = \max\{c^T x \mid Ax = b, x \ge 0\}$$
$$w = \min\{b^T y \mid A^T y \ge c\}$$

not have non-negativity constraints for the dual variables.

This means for computing the dual of a standard form LP, we do

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$$\max\{c^{T}x \mid Ax = b, x \ge 0\}$$

$$= \max\{c^{T}x \mid Ax \le b, -Ax \le -b, x \ge 0\}$$

$$= \max\{c^{T}x \mid \begin{bmatrix} A \\ -A \end{bmatrix} x \le \begin{bmatrix} b \\ -b \end{bmatrix}, x \ge 0\}$$

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$$\min\{[b^T - b^T]y \mid [A^T - A^T]y \ge c, y \ge 0\}$$

$$= \min\left\{[b^T - b^T] \cdot \begin{bmatrix} y^+ \\ y^- \end{bmatrix} \mid [A^T - A^T] \cdot \begin{bmatrix} y^+ \\ y^- \end{bmatrix} \ge c, y^- \ge 0, y^+ \ge 0\right\}$$

 $= \min \left\{ b^T \cdot (y^+ - y^-) \mid A^T \cdot (y^+ - y^-) \ge c, y^- \ge 0, y^+ \ge 0 \right\}$

5.2 Simplex and Duality

$$z = \max\{c^T x \mid Ax = b, x \ge 0\}$$

$$w = \min\{b^T y \mid A^T y > c\}$$

The following linear programs form a primal dual pair:

not have non-negativity constraints for the dual variables.

This means for computing the dual of a standard form LP, we do

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EADS II 5.2 Simplex and Duality

$$\max\{c^T x \mid Ax = b, x \ge 0\}$$

$$= \max\{c^T x \mid Ax \le b, -Ax \le -b, x \ge 0\}$$

$$= \max\{c^T x \mid \begin{bmatrix} A \\ -A \end{bmatrix} x \le \begin{bmatrix} b \\ -b \end{bmatrix}, x \ge 0\}$$

Dual:

Dual:

$$\min\{[b^T - b^T]y \mid [A^T - A^T]y \ge c, y \ge 0\}$$

$$= \min\{[h^T - h^T], [y^+] \mid [A^T - A^T], [y^+]\}$$

$$\begin{bmatrix} y^T - b^T \end{bmatrix} \cdot \begin{bmatrix} y^+ \\ y^- \end{bmatrix} \begin{bmatrix} A^T - A^T \end{bmatrix}$$

$$\begin{bmatrix} T & -b^T \end{bmatrix} \cdot \begin{bmatrix} \mathcal{Y}^+ \\ \mathcal{Y}^- \end{bmatrix} \mid [A^T - A^T]$$

$$= \min \left\{ \begin{bmatrix} b^T - b^T \end{bmatrix} \cdot \begin{bmatrix} y^+ \\ y^- \end{bmatrix} \middle| \begin{bmatrix} A^T - A^T \end{bmatrix} \cdot \begin{bmatrix} y^+ \\ y^- \end{bmatrix} \ge c, y^- \ge 0, y^+ \ge 0 \right\}$$

$$-b^{T}] \cdot \begin{bmatrix} y \\ y^{-} \end{bmatrix} \mid [A^{T} - A^{T}] \cdot \begin{bmatrix} y \\ y^{-} \end{bmatrix} \ge c, y^{-} \ge 0, y^{+}$$

$$= \min \left\{ \begin{bmatrix} b^{T} - b^{T} \end{bmatrix} \cdot \begin{bmatrix} y \\ y^{-} \end{bmatrix} \mid \begin{bmatrix} A^{T} - A^{T} \end{bmatrix} \cdot \begin{bmatrix} y \\ y^{-} \end{bmatrix} \ge c, y^{-} \ge 0, y^{+} \ge 0 \right\}$$

$$= \min \left\{ b^{T} \cdot (y^{+} - y^{-}) \mid A^{T} \cdot (y^{+} - y^{-}) \ge c, y^{-} \ge 0, y^{+} \ge 0 \right\}$$

5.2 Simplex and Duality

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$$z = \max\{c^T x \mid Ax = b, x \ge 0\}$$
$$w = \min\{b^T y \mid A^T y \ge c\}$$

This means for computing the dual of a standard form LP, we do

 $= \min \left\{ b^T y' \mid A^T y' \ge c \right\}$

Suppose that we have a basic feasible solution with reduced cost

$$\tilde{c} = c^T - c_B^T A_B^{-1} A \le 0$$

$$y^* = (A_B^{-1})^T c_B$$
 is solution to the dual $\min\{b^T y | A^T y \ge c\}$.

Proof

Primal:

$$\max\{c^{T}x \mid Ax = b, x \ge 0\}$$

$$= \max\{c^{T}x \mid Ax \le b, -Ax \le -b, x \ge 0\}$$

$$= \max\{c^{T}x \mid \begin{bmatrix} A \\ -A \end{bmatrix} x \le \begin{bmatrix} b \\ -b \end{bmatrix}, x \ge 0\}$$

Dual:

$$\min\{ \begin{bmatrix} b^T - b^T \end{bmatrix} y \mid \begin{bmatrix} A^T - A^T \end{bmatrix} y \ge c, y \ge 0 \}$$

$$= \min\left\{ \begin{bmatrix} b^T - b^T \end{bmatrix} \cdot \begin{bmatrix} y^+ \\ y^- \end{bmatrix} \mid \begin{bmatrix} A^T - A^T \end{bmatrix} \cdot \begin{bmatrix} y^+ \\ y^- \end{bmatrix} \ge c, y^- \ge 0, y^+ \ge 0 \right\}$$

$$= \min\left\{ b^T \cdot (y^+ - y^-) \mid A^T \cdot (y^+ - y^-) \ge c, y^- \ge 0, y^+ \ge 0 \right\}$$

$$= \min\left\{ b^T y' \mid A^T y' \ge c \right\}$$

5.2 Simplex and Duality

Suppose that we have a basic feasible solution with reduced cost

$$\tilde{c} = c^T - c_B^T A_B^{-1} A \le 0$$

This is equivalent to $A^T(A_R^{-1})^T c_R \ge c$

$$y^* = (A_B^{-1})^T c_B$$
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Proof

Primal:

$$\max\{c^{T}x \mid Ax = b, x \ge 0\}$$

$$= \max\{c^{T}x \mid Ax \le b, -Ax \le -b, x \ge 0\}$$

$$= \max\{c^{T}x \mid \begin{bmatrix} A \\ -A \end{bmatrix} x \le \begin{bmatrix} b \\ -b \end{bmatrix}, x \ge 0\}$$

Dual:

$$\min\{ \begin{bmatrix} b^T - b^T \end{bmatrix} y \mid \begin{bmatrix} A^T - A^T \end{bmatrix} y \ge c, y \ge 0 \}$$

$$= \min\left\{ \begin{bmatrix} b^T - b^T \end{bmatrix} \cdot \begin{bmatrix} y^+ \\ y^- \end{bmatrix} \mid \begin{bmatrix} A^T - A^T \end{bmatrix} \cdot \begin{bmatrix} y^+ \\ y^- \end{bmatrix} \ge c, y^- \ge 0, y^+ \ge 0 \right\}$$

$$= \min\left\{ b^T \cdot (y^+ - y^-) \mid A^T \cdot (y^+ - y^-) \ge c, y^- \ge 0, y^+ \ge 0 \right\}$$

$$= \min\left\{ b^T y' \mid A^T y' \ge c \right\}$$

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$$\tilde{c} = c^T - c_B^T A_B^{-1} A \le 0$$

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$$v^* = (A_R^{-1})^T c_R$$
 is solution to the dual $\min\{b^T v | A^T v \ge c\}$.

$$b^{T}y^{*} = (Ax^{*})^{T}y^{*} = (A_{B}x_{B}^{*})^{T}y^{*}$$
$$= (A_{B}x_{B}^{*})^{T}(A_{B}^{-1})^{T}c_{B} = (x_{B}^{*})^{T}A_{B}^{T}(A_{B}^{-1})^{T}c_{B}$$
$$= c^{T}x^{*}$$

Proof

Primal:

$$\max\{c^{T}x \mid Ax = b, x \ge 0\}$$

$$= \max\{c^{T}x \mid Ax \le b, -Ax \le -b, x \ge 0\}$$

$$= \max\{c^{T}x \mid \begin{bmatrix} A \\ -A \end{bmatrix} x \le \begin{bmatrix} b \\ -b \end{bmatrix}, x \ge 0\}$$

Dual:

$$\min\{ [b^{T} - b^{T}]y \mid [A^{T} - A^{T}]y \geq c, y \geq 0 \}$$

$$= \min \left\{ [b^{T} - b^{T}] \cdot \begin{bmatrix} y^{+} \\ y^{-} \end{bmatrix} \mid [A^{T} - A^{T}] \cdot \begin{bmatrix} y^{+} \\ y^{-} \end{bmatrix} \geq c, y^{-} \geq 0, y^{+} \geq 0 \right\}$$

$$= \min \left\{ b^{T} \cdot (y^{+} - y^{-}) \mid A^{T} \cdot (y^{+} - y^{-}) \geq c, y^{-} \geq 0, y^{+} \geq 0 \right\}$$

$$= \min \left\{ b^{T}y' \mid A^{T}y' \geq c \right\}$$

5.2 Simplex and Duality

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$$\tilde{c} = c^T - c_B^T A_B^{-1} A \le 0$$

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Proof

Primal:

$$\max\{c^{T}x \mid Ax = b, x \ge 0\}$$

$$= \max\{c^{T}x \mid Ax \le b, -Ax \le -b, x \ge 0\}$$

$$= \max\{c^{T}x \mid \begin{bmatrix} A \\ -A \end{bmatrix} x \le \begin{bmatrix} b \\ -b \end{bmatrix}, x \ge 0\}$$

Dual:

$$\min\{ [b^{T} - b^{T}]y \mid [A^{T} - A^{T}]y \geq c, y \geq 0 \}$$

$$= \min \left\{ [b^{T} - b^{T}] \cdot \begin{bmatrix} y^{+} \\ y^{-} \end{bmatrix} \mid [A^{T} - A^{T}] \cdot \begin{bmatrix} y^{+} \\ y^{-} \end{bmatrix} \geq c, y^{-} \geq 0, y^{+} \geq 0 \right\}$$

$$= \min \left\{ b^{T} \cdot (y^{+} - y^{-}) \mid A^{T} \cdot (y^{+} - y^{-}) \geq c, y^{-} \geq 0, y^{+} \geq 0 \right\}$$

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5.2 Simplex and Duality

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Proof

Primal:

$$\max\{c^{T}x \mid Ax = b, x \ge 0\}$$

$$= \max\{c^{T}x \mid Ax \le b, -Ax \le -b, x \ge 0\}$$

$$= \max\{c^{T}x \mid \begin{bmatrix} A \\ -A \end{bmatrix} x \le \begin{bmatrix} b \\ -b \end{bmatrix}, x \ge 0\}$$

Dual:

$$\min\{ [b^{T} - b^{T}] y \mid [A^{T} - A^{T}] y \ge c, y \ge 0 \}$$

$$= \min \left\{ [b^{T} - b^{T}] \cdot \begin{bmatrix} y^{+} \\ y^{-} \end{bmatrix} \mid [A^{T} - A^{T}] \cdot \begin{bmatrix} y^{+} \\ y^{-} \end{bmatrix} \ge c, y^{-} \ge 0, y^{+} \ge 0 \right\}$$

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$$= c^{T}y^{*}$$

Proof

Primal:

$$\max\{c^{T}x \mid Ax = b, x \ge 0\}$$

$$= \max\{c^{T}x \mid Ax \le b, -Ax \le -b, x \ge 0\}$$

$$= \max\{c^{T}x \mid \begin{bmatrix} A \\ -A \end{bmatrix} x \le \begin{bmatrix} b \\ -b \end{bmatrix}, x \ge 0\}$$

Dual:

$$\min\{ \begin{bmatrix} b^T - b^T \end{bmatrix} y \mid \begin{bmatrix} A^T - A^T \end{bmatrix} y \ge c, y \ge 0 \}$$

$$= \min\left\{ \begin{bmatrix} b^T - b^T \end{bmatrix} \cdot \begin{bmatrix} y^+ \\ y^- \end{bmatrix} \mid \begin{bmatrix} A^T - A^T \end{bmatrix} \cdot \begin{bmatrix} y^+ \\ y^- \end{bmatrix} \ge c, y^- \ge 0, y^+ \ge 0 \right\}$$

$$= \min\left\{ b^T \cdot (y^+ - y^-) \mid A^T \cdot (y^+ - y^-) \ge c, y^- \ge 0, y^+ \ge 0 \right\}$$

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$$= (A_{B}x_{B}^{*})^{T}(A_{B}^{-1})^{T}c_{B} = (x_{B}^{*})^{T}A_{B}^{T}(A_{B}^{-1})^{T}c_{B}$$

Proof

Primal:

$$\max\{c^{T}x \mid Ax = b, x \ge 0\}$$

$$= \max\{c^{T}x \mid Ax \le b, -Ax \le -b, x \ge 0\}$$

$$= \max\{c^{T}x \mid \begin{bmatrix} A \\ -A \end{bmatrix} x \le \begin{bmatrix} b \\ -b \end{bmatrix}, x \ge 0\}$$

Dual:

$$\min\{ [b^{T} - b^{T}]y \mid [A^{T} - A^{T}]y \ge c, y \ge 0 \}$$

$$= \min \left\{ [b^{T} - b^{T}] \cdot \begin{bmatrix} y^{+} \\ y^{-} \end{bmatrix} \mid [A^{T} - A^{T}] \cdot \begin{bmatrix} y^{+} \\ y^{-} \end{bmatrix} \ge c, y^{-} \ge 0, y^{+} \ge 0 \right\}$$

$$= \min \left\{ b^{T} \cdot (y^{+} - y^{-}) \mid A^{T} \cdot (y^{+} - y^{-}) \ge c, y^{-} \ge 0, y^{+} \ge 0 \right\}$$

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5.2 Simplex and Duality

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$$= (A_{B}x_{B}^{*})^{T}(A_{B}^{-1})^{T}c_{B} = (x_{B}^{*})^{T}A_{B}^{T}(A_{B}^{-1})^{T}c_{B}$$
$$= c^{T}x^{*}$$

Proof

Primal:

$$\max\{c^{T}x \mid Ax = b, x \ge 0\}$$

$$= \max\{c^{T}x \mid Ax \le b, -Ax \le -b, x \ge 0\}$$

$$= \max\{c^{T}x \mid \begin{bmatrix} A \\ -A \end{bmatrix} x \le \begin{bmatrix} b \\ -b \end{bmatrix}, x \ge 0\}$$

Dual:

$$\min\{ \begin{bmatrix} b^T - b^T \end{bmatrix} y \mid \begin{bmatrix} A^T - A^T \end{bmatrix} y \ge c, y \ge 0 \}$$

$$= \min\left\{ \begin{bmatrix} b^T - b^T \end{bmatrix} \cdot \begin{bmatrix} y^+ \\ y^- \end{bmatrix} \mid \begin{bmatrix} A^T - A^T \end{bmatrix} \cdot \begin{bmatrix} y^+ \\ y^- \end{bmatrix} \ge c, y^- \ge 0, y^+ \ge 0 \right\}$$

$$= \min\left\{ b^T \cdot (y^+ - y^-) \mid A^T \cdot (y^+ - y^-) \ge c, y^- \ge 0, y^+ \ge 0 \right\}$$

$$= \min\left\{ b^T y' \mid A^T y' \ge c \right\}$$

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$$= (A_{B}x_{B}^{*})^{T}(A_{B}^{-1})^{T}c_{B} = (x_{B}^{*})^{T}A_{B}^{T}(A_{B}^{-1})^{T}c_{B}$$
$$= c^{T}x^{*}$$

Hence, the solution is optimal.

Proof

Primal:

$$\max\{c^{T}x \mid Ax = b, x \ge 0\}$$

$$= \max\{c^{T}x \mid Ax \le b, -Ax \le -b, x \ge 0\}$$

$$= \max\{c^{T}x \mid \begin{bmatrix} A \\ -A \end{bmatrix} x \le \begin{bmatrix} b \\ -b \end{bmatrix}, x \ge 0\}$$

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$$= \min\left\{ b^T \cdot (y^+ - y^-) \mid A^T \cdot (y^+ - y^-) \ge c, y^- \ge 0, y^+ \ge 0 \right\}$$

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