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Lemma 2

Assume a linear program $P = \max\{c^T x \mid Ax \le b; x \ge 0\}$ has solution x^* and its dual $D = \min\{b^T y \mid A^T y \ge c; y \ge 0\}$ has solution y^*.

1. *If $x_j^* > 0$ then the j-th constraint in D is tight.*
2. *If the j-th constraint in D is not tight than $x_j^* = 0$.*
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If we say that a variable x_j^* (y_i^*) has slack if $x_j^* > 0$ ($y_i^* > 0$), (i.e., the corresponding variable restriction is not tight) and a contraint has slack if it is not tight, then the above says that for a primal-dual solution pair it is not possible that a constraint **and** its corresponding (dual) variable has slack.

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Analogous to the proof of weak duality we obtain

$$c^T x^* \le y^{*T} A x^* \le b^T y^*$$

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- Brewer: find mix of ale and beer that maximizes profits

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\begin{array}{lrcrl}
\max & 13a & + & 23b & \\
\text{s.t.} & 5a & + & 15b & \leq 480 \\
 & 4a & + & 4b & \leq 160 \\
 & 35a & + & 20b & \leq 1190 \\
 & & & a,b & \geq 0
\end{array}
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- Entrepeneur: buy resources from brewer at minimum cost
  C, H, M: unit price for corn, hops and malt.

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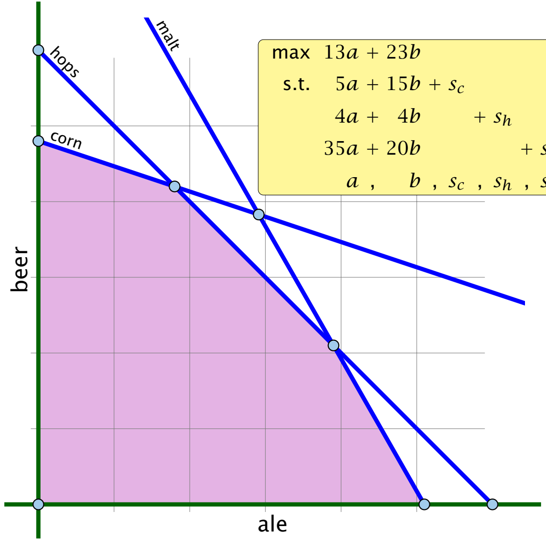

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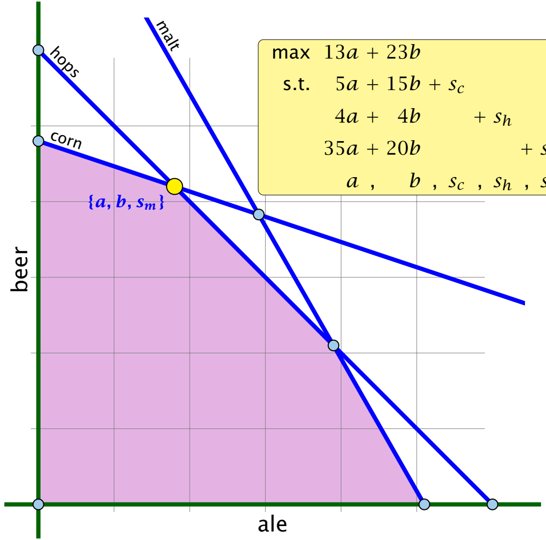

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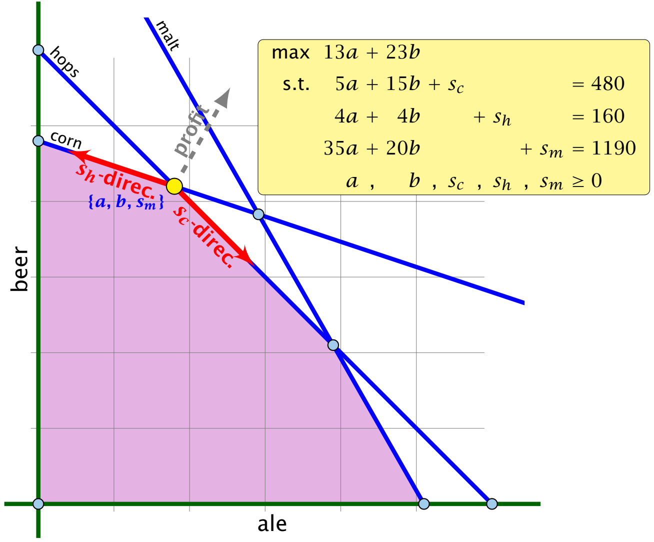

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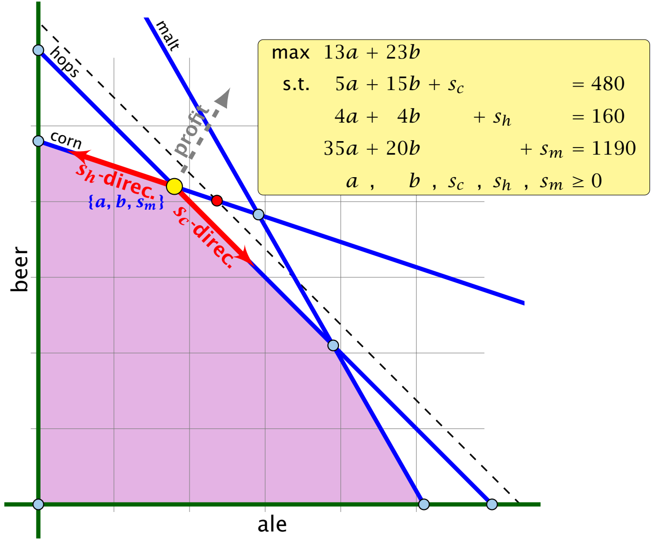

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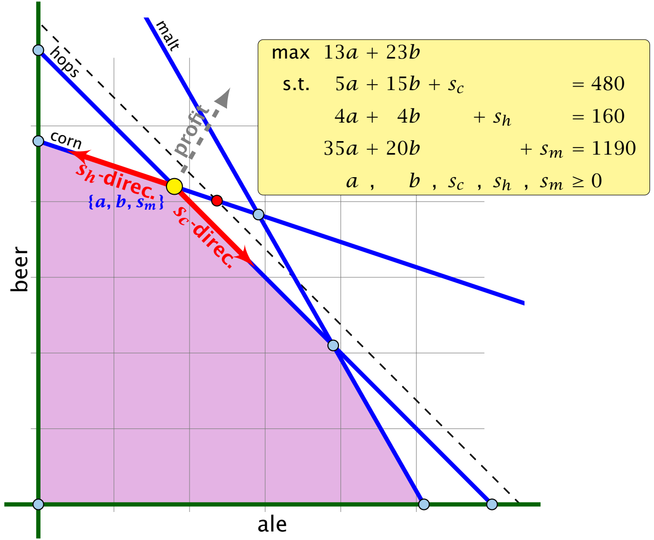

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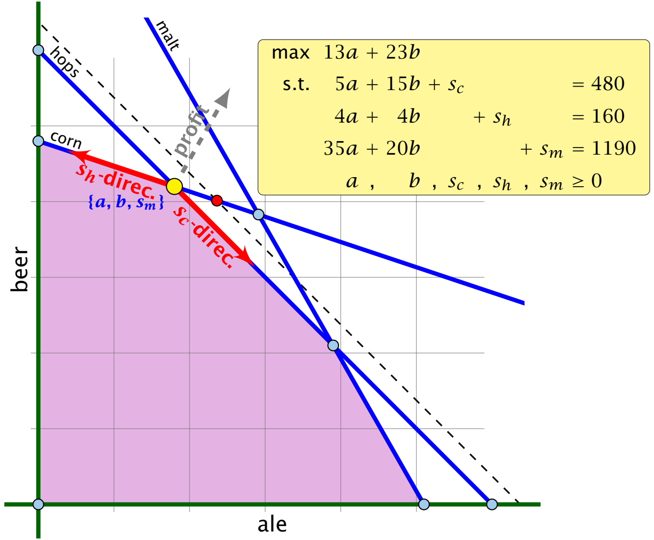
If ϵ is “small” enough then the optimum dual solution y^* might not change. Therefore the profit increases by $\sum_i \varepsilon_i y_i^*$.

Therefore we can interpret the dual variables as marginal prices.

Note that with this interpretation, complementary slackness becomes obvious.

- If the brewer has slack of some resource (e.g. corn) then he is not willing to pay anything for it (corresponding dual variable is zero).
- If the dual variable for some resource is non-zero, then an increase of this resource increases the profit of the brewer. Hence, it makes no sense to have left-overs of this resource. Therefore its slack must be zero.

Example



The change in profit when increasing hops by one unit is $= c_B^T A_B^{-1} e_h$.

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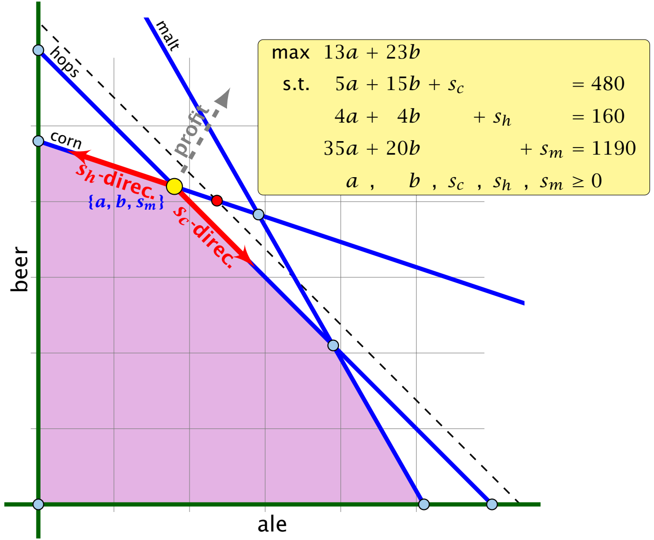

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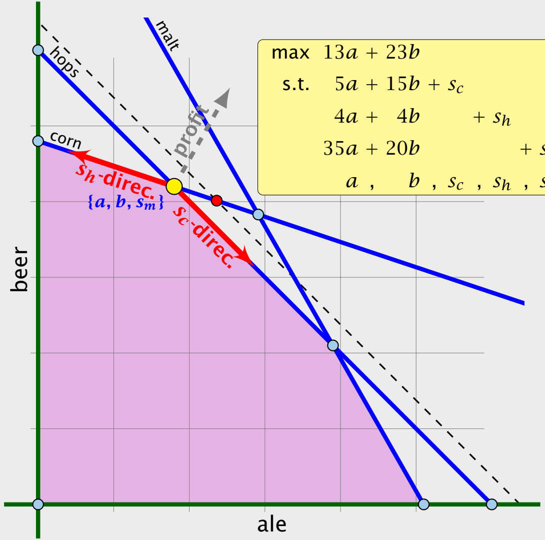
Of course, the previous argument about the increase in the primal objective only holds for the non-degenerate case.

If the optimum basis is degenerate then increasing the supply of one resource may not allow the objective value to increase.

Example



$$
\begin{array}{llllllll}
\text{max} & 13a & + & 23b & & & & \\
\text{s.t.} & 5a & + & 15b & + s_c & & & = 480 \\
 & 4a & + & 4b & & + s_h & & = 160 \\
 & 35a & + & 20b & & & + s_m & = 1190 \\
 & a & , & b & , s_c & , s_h & , s_m & \geq 0
\end{array}
$$

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