19 Cuts & Metrics

Shortest Path

$$\begin{array}{llrcl} \min & & \sum_e c(e) x_e & & \\ \text{s.t.} & \forall S \in \mathcal{S} & \sum_{e \in \delta(S)} x_e & \geq & 1 \\ & \forall e \in E & x_e & \in & \{0,1\} \end{array}$$

$\mathcal{S}$ is the set of subsets that separate s from t.

The Dual:

$$\begin{array}{llrcl} \max & & \sum_S y_S & & \\ \text{s.t.} & \forall e \in E & \sum_{S: e \in \delta(S)} y_S & \leq & c(e) \\ & \forall S \in \mathcal{S} & y_S & \geq & 0 \end{array}$$

The Separation Problem for the Shortest Path LP is the Minimum Cut Problem.

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How do we round the LP?

- Let $B(s,r)$ be the ball of radius r around s (w.r.t. metric d). Formally:
$$B = \{v \in V \mid d(s,v) \le r\}$$
- For $0 \le r < 1$, $B(s,r)$ is an s-t-cut.

Which value of r should we choose? choose randomly!!!

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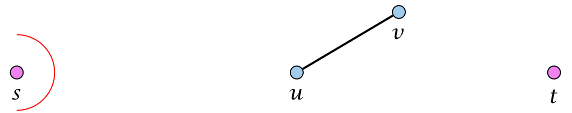

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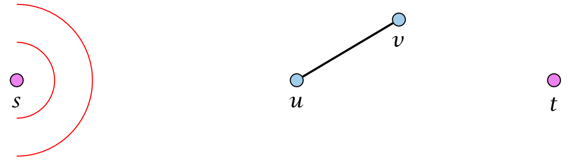

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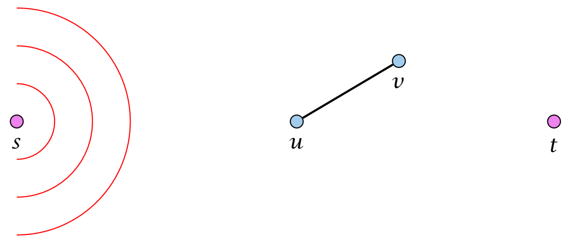

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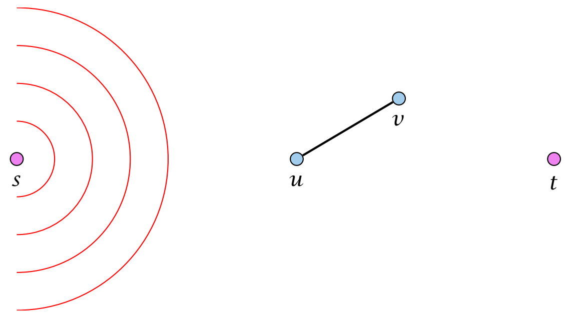

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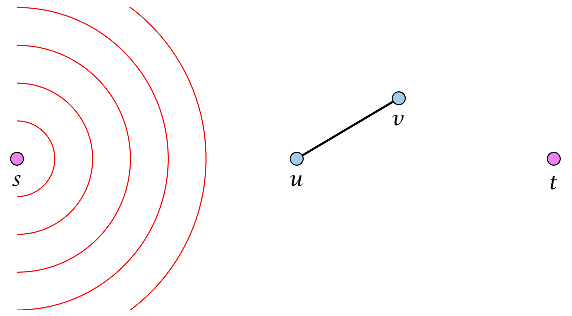

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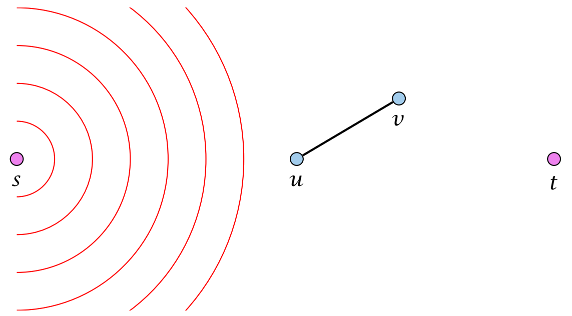

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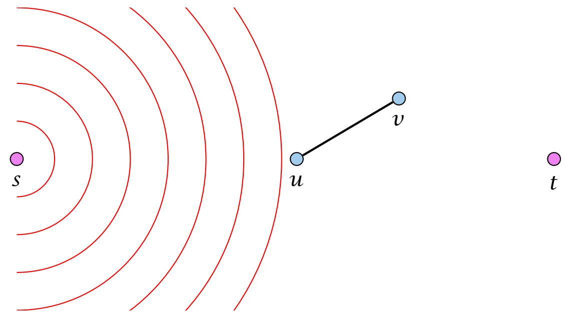

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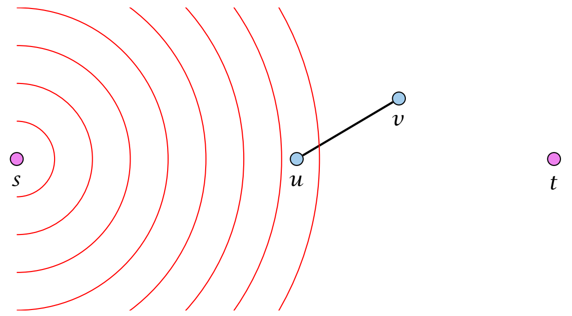

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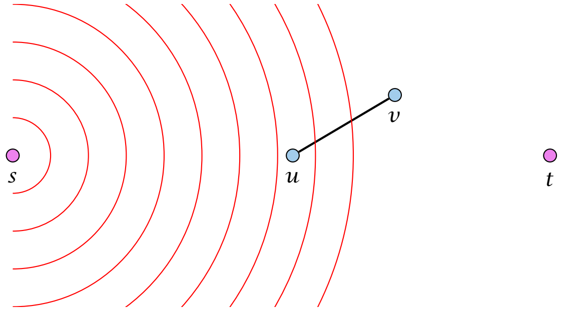

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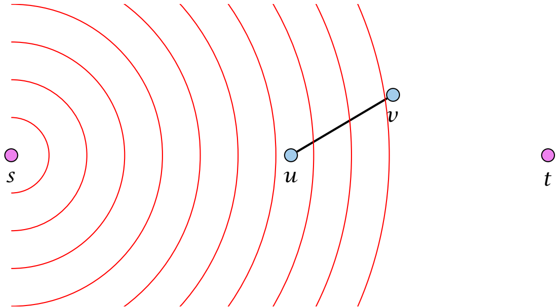

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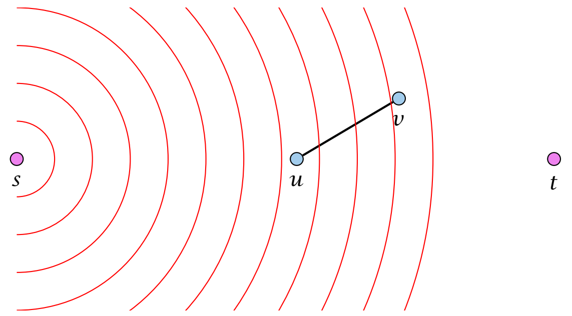

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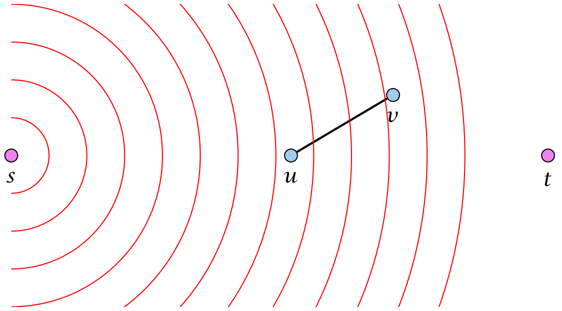

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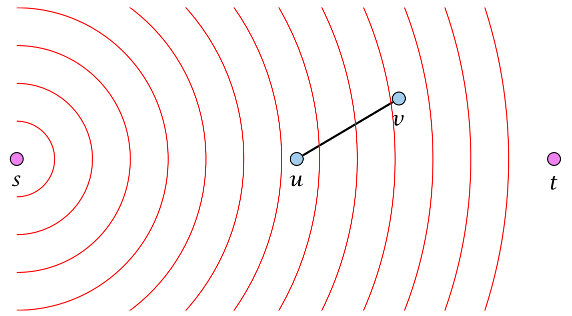

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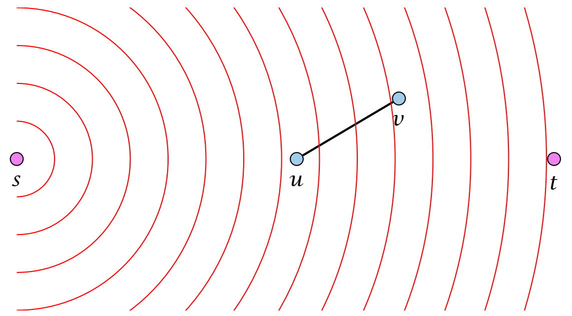

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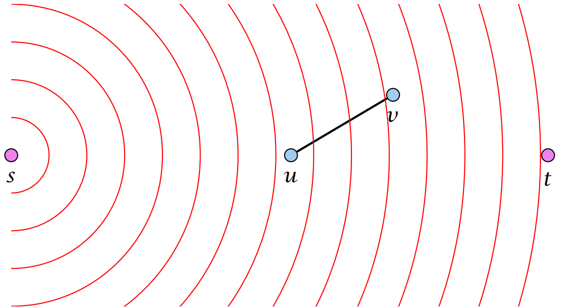

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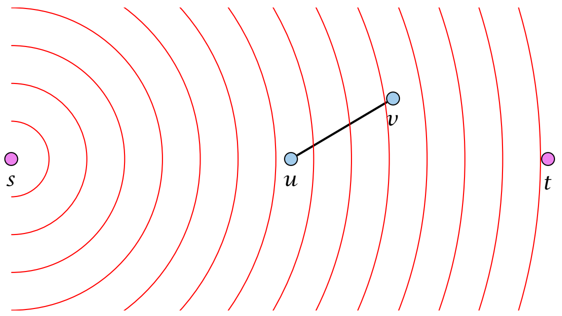

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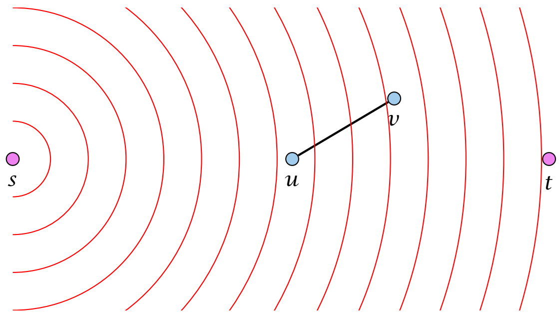

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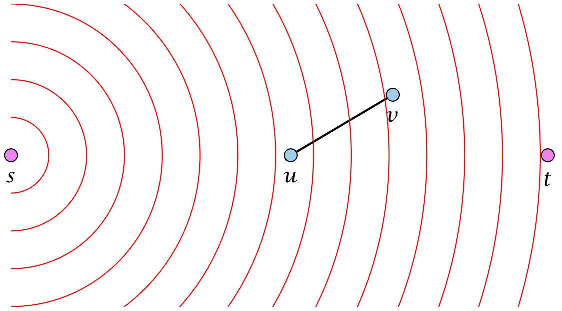

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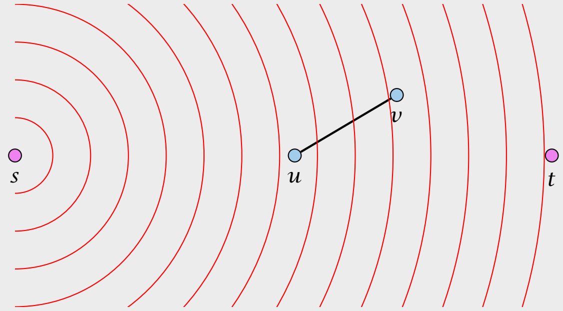

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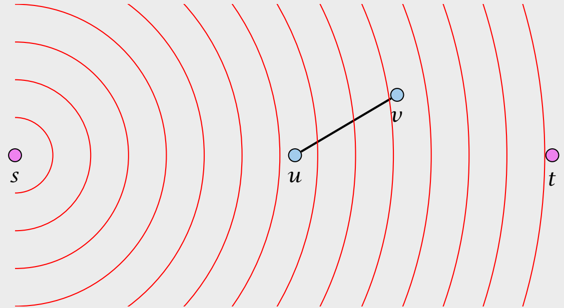

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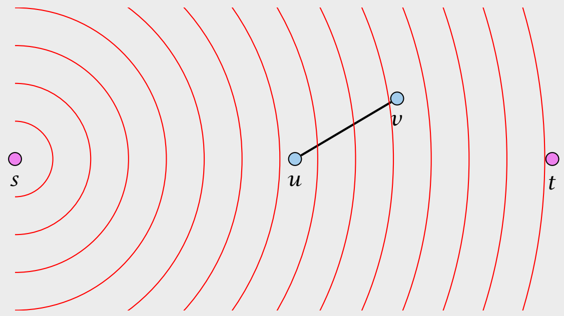

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Algorithm 1 Multicut(G')

while $\exists s_i$-t_i pair in G' **do**
    $C \leftarrow$ RegionGrowing(s_i, p)
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- probability of cutting an edge is only p
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Problem:
We may not cut all source-target pairs.

A component that we remove may contain an s_i-t_i pair.

If we ensure that we cut before reaching radius $1/2$ we are in good shape.

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while $\exists s_i$-t_i pair in G' **do**
    $C \leftarrow$ RegionGrowing(s_i, p)
    $G' = G' \setminus C$ // cuts edges leaving C
return $B(s_i, z)$

- probability of cutting an edge is only p
- a source either does not reach an edge during Region Growing; then it is not cut
- if it reaches the edge then it either cuts the edge or protects the edge from being cut by other sources
- if we choose $p = \delta$ the probability of cutting an edge is only its LP-value; our expected cost are at most OPT.

Problem:

We may not cut all source-target pairs.

A component that we remove may contain an s_i-t_i pair.

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- choose $p = 6\ln k \cdot \delta$
- we make $\frac{1}{2\delta}$ trials before reaching radius $1/2$.
- we say a Region Growing is not successful if it does not terminate before reaching radius $1/2$.

$$\Pr[\text{not successful}] \le (1-p)^{\frac{1}{2\delta}} = \left((1-p)^{1/p}\right)^{\frac{p}{2\delta}} \le e^{-\frac{p}{2\delta}} \le \frac{1}{k^3}$$

- Hence,

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What is expected cost?

$$\begin{aligned}\mathrm{E}[\text{cutsize}] &= \Pr[\text{success}] \cdot \mathrm{E}[\text{cutsize} \mid \text{success}] \\ &\quad + \Pr[\text{no success}] \cdot \mathrm{E}[\text{cutsize} \mid \text{no success}]\end{aligned}$$

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$$\mathrm{E}[\text{cutsize}] = \Pr[\text{success}] \cdot \mathrm{E}[\text{cutsize} \mid \text{success}] + \Pr[\text{no success}] \cdot \mathrm{E}[\text{cutsize} \mid \text{no success}]$$

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If we are not successful we simply perform a trivial k-approximation.

This only increases the expected cost by at most $\frac{1}{k^2} \cdot k\mathrm{OPT} \leq \mathrm{OPT}/k$.

Hence, our final cost is $\mathcal{O}(\ln k) \cdot \mathrm{OPT}$ in expectation.

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