8 Anwendung: Suchen

Gegeben: Folge `a` ganzer Zahlen; Element `x`

Gesucht: Wo kommt `x` in `a` vor?

Naives Vorgehen:

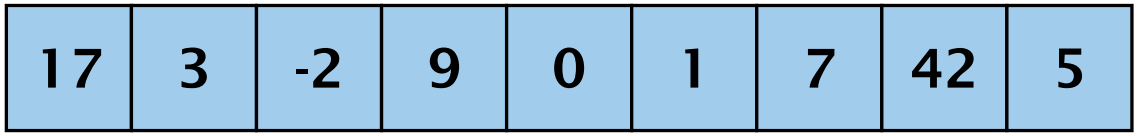
- Vergleiche `x` der Reihe nach mit `a[0]`, `a[1]`, usw.
- Finden wir `i` mit `a[i] == x`, geben wir `i` aus.
- Andernfalls geben wir `-1` aus: „Element nicht gefunden“!

Naives Suchen

```
1 public static int find(int[] a, int x) {
2     int i = 0;
3     while (i < a.length && a[i] != x)
4         ++i;
5     if (i == a.length)
6         return -1;
7     else
8         return i;
9 }
```

Naives Suchen

Beispiel



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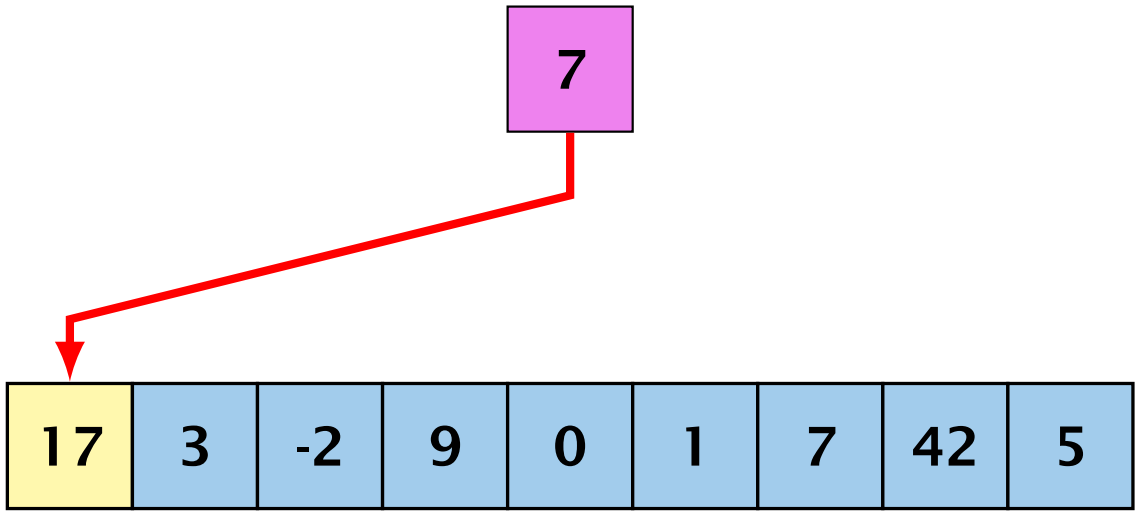
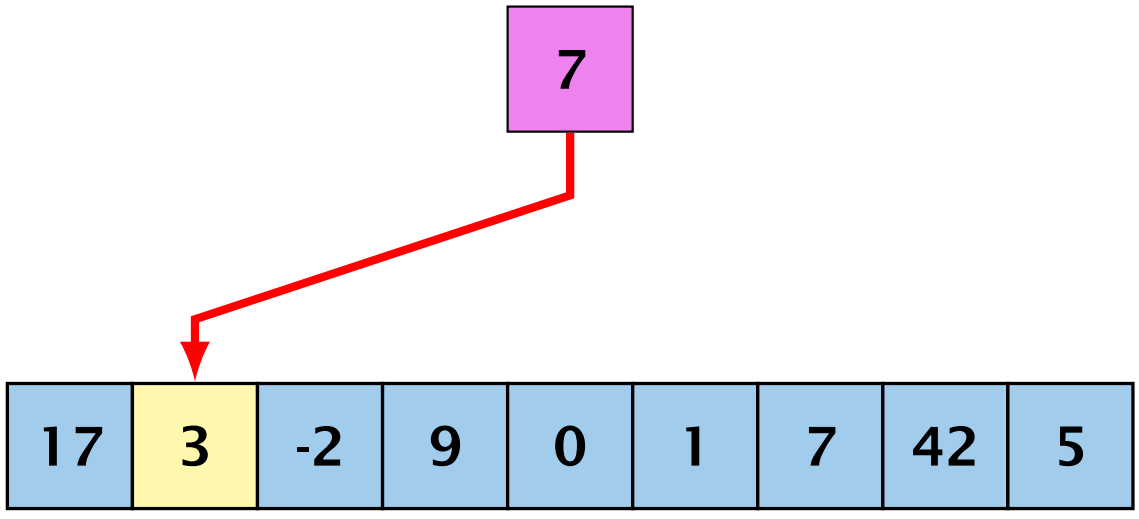


no

Beispiel



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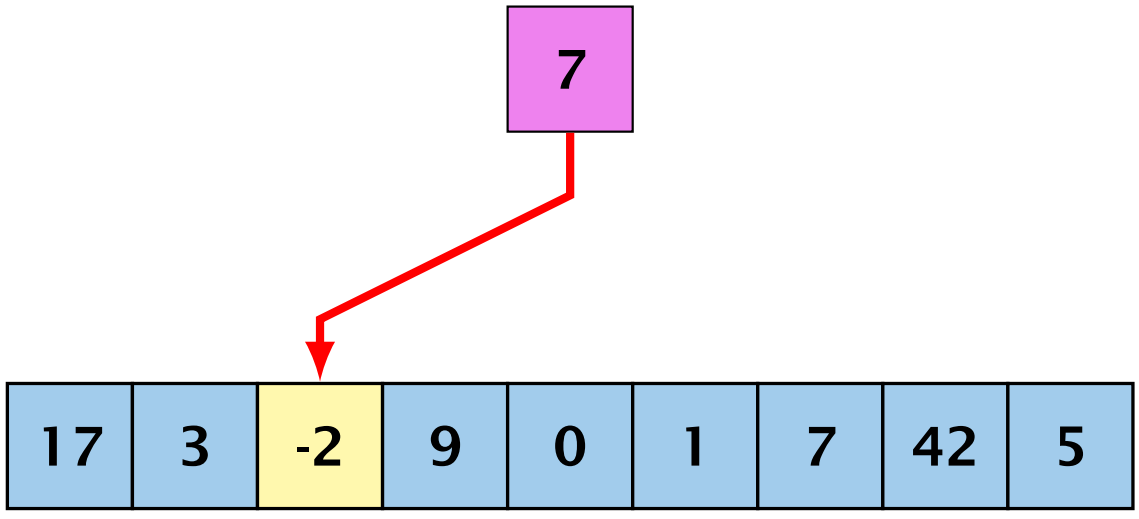

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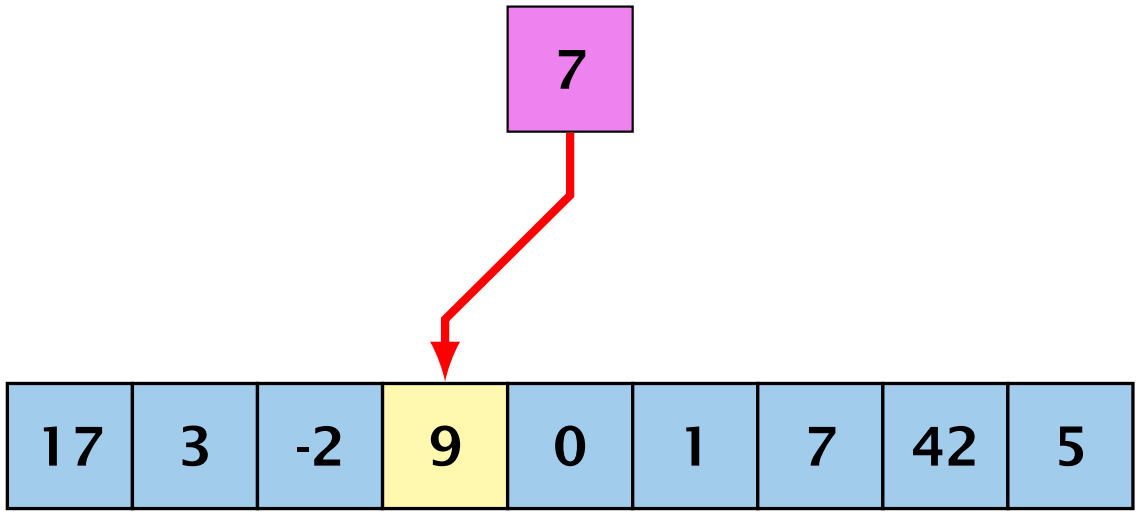

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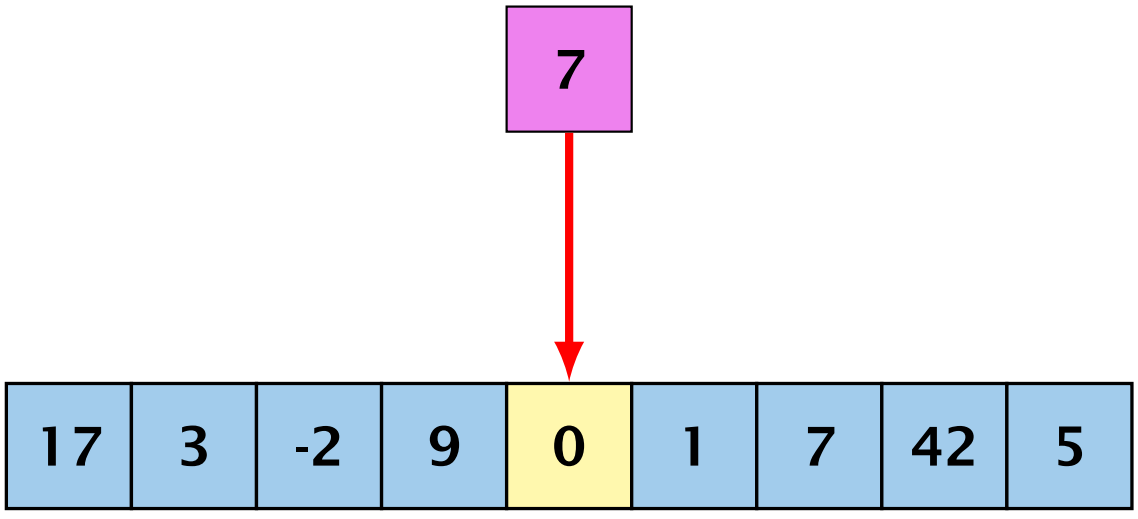

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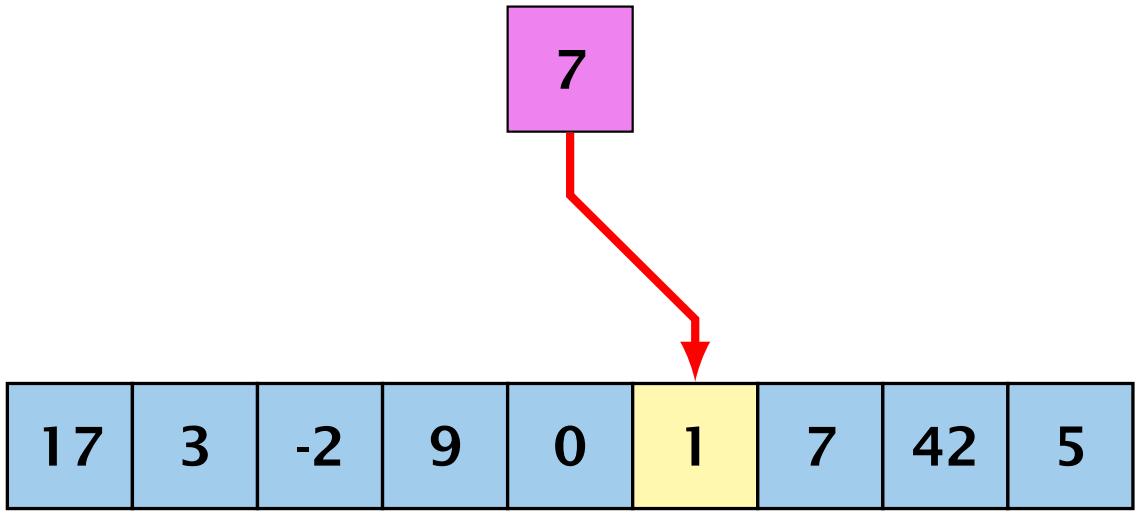
Beispiel



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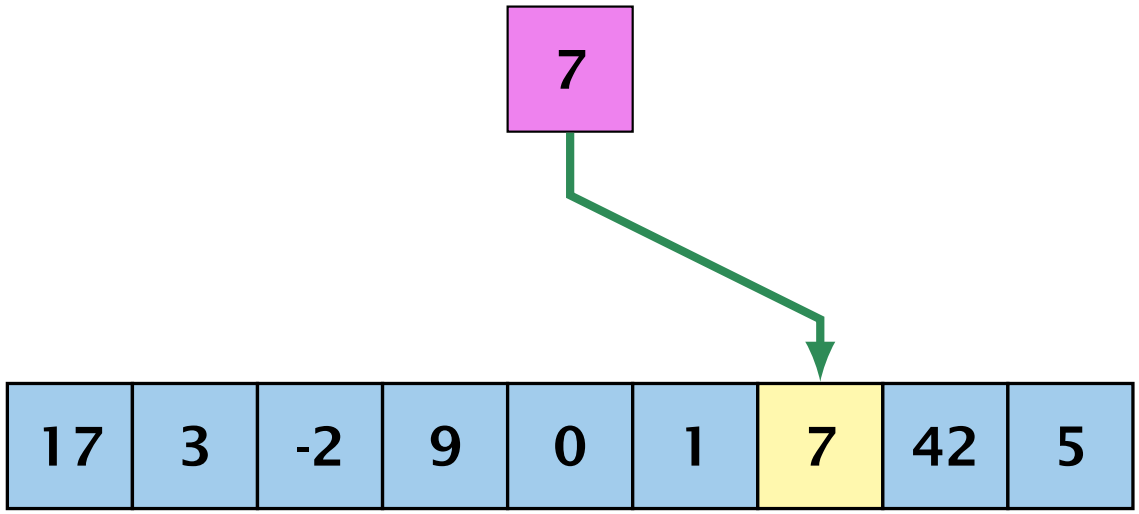


yes

Naives Suchen

- Im Beispiel benötigen wir 7 Vergleiche
- Im schlimmsten Fall (worst case) benötigen wir bei einem Feld der Länge n sogar n Vergleiche.
- Kommt x tatsächlich im Feld vor, benötigen wir selbst im Durchschnitt $(n+1)/2$ Vergleiche.

...geht das nicht besser?

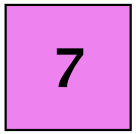
Binäre Suche

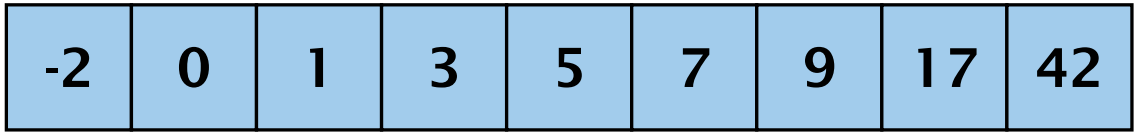
Idee:

- Sortiere das Feld.
- Vergleiche x mit dem Wert, der in der Mitte steht.
- Liegt Gleichheit vor, sind wir fertig.
- Ist x kleiner, brauchen wir nur noch links weitersuchen.
- Ist x größer, brauchen wir nur noch rechts weiter suchen.

$\Longrightarrow$ binäre Suche

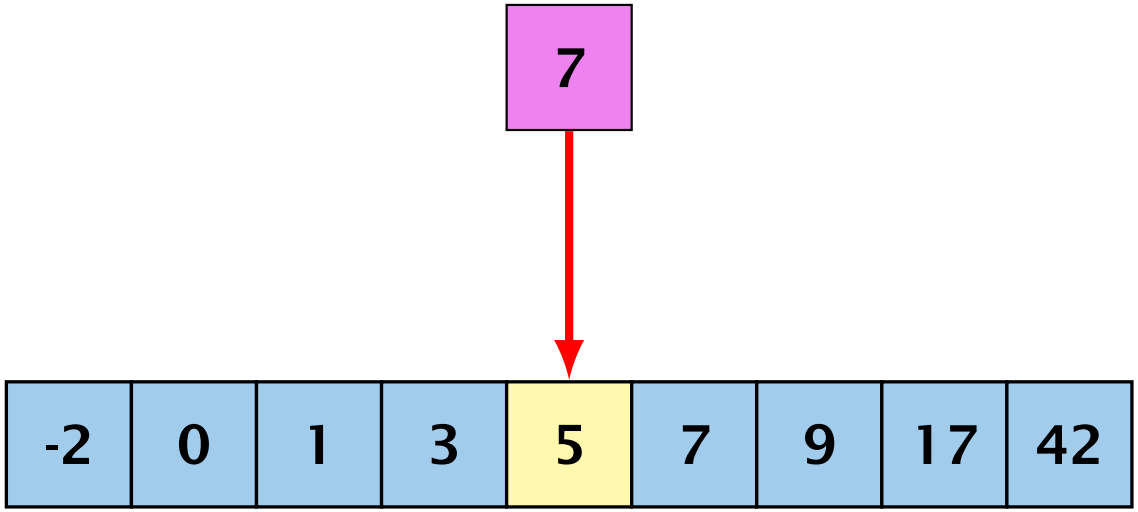
Beispiel





- wir benötigen nur drei Vergleiche
- hat das Feld $2^n - 1$ Elemente, benötigen wir maximal n Vergleiche

Beispiel



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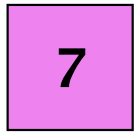

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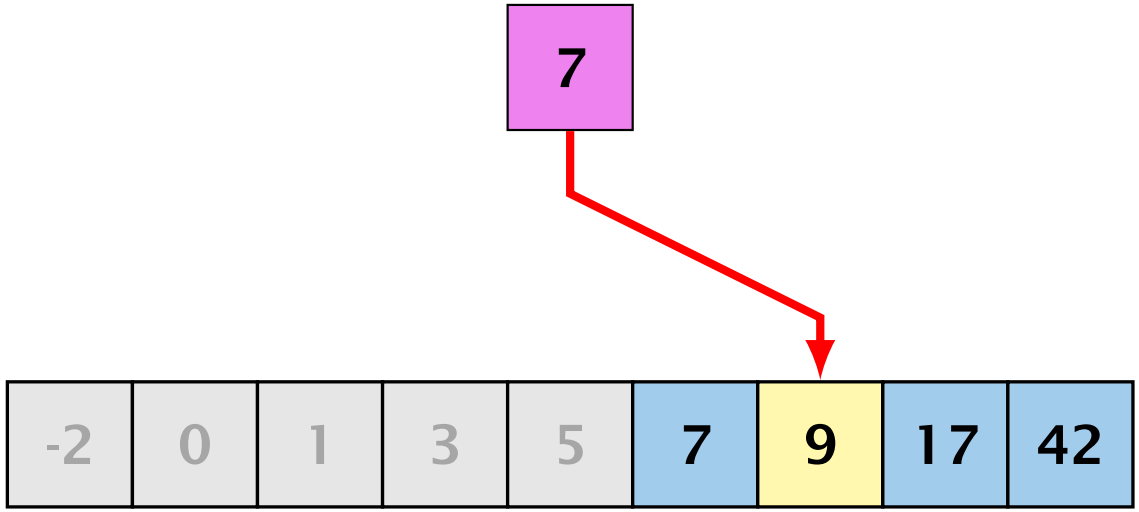

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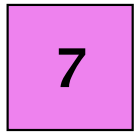

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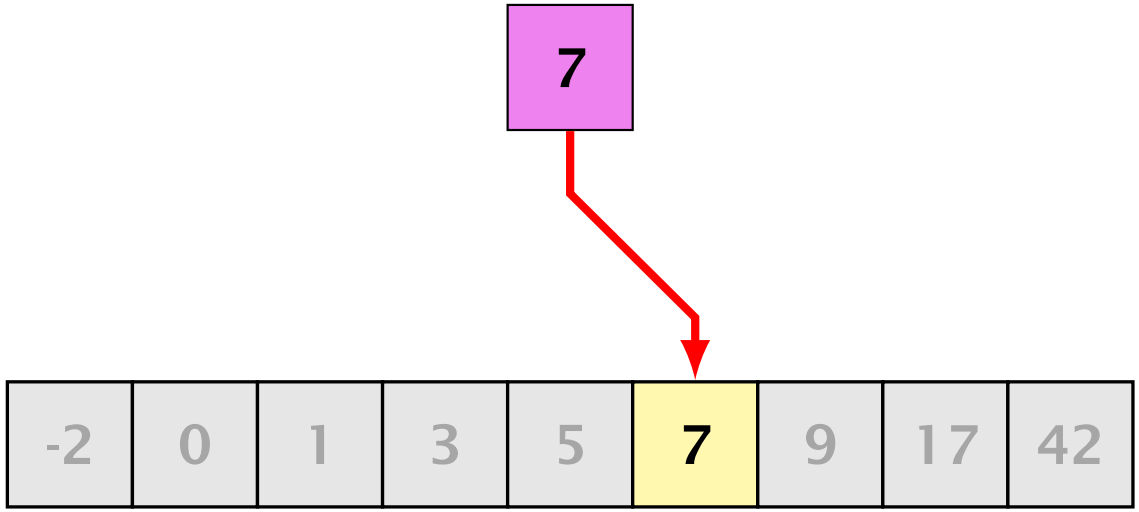

yes

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- hat das Feld $2^n - 1$ Elemente, benötigen wir maximal n Vergleiche

Implementierung

Idee:

Führe Hilfsfunktion

```
public static int find0(int[] a,int x,int n1,int n2)
```

ein, die im Interval $[n1, n2]$ sucht.

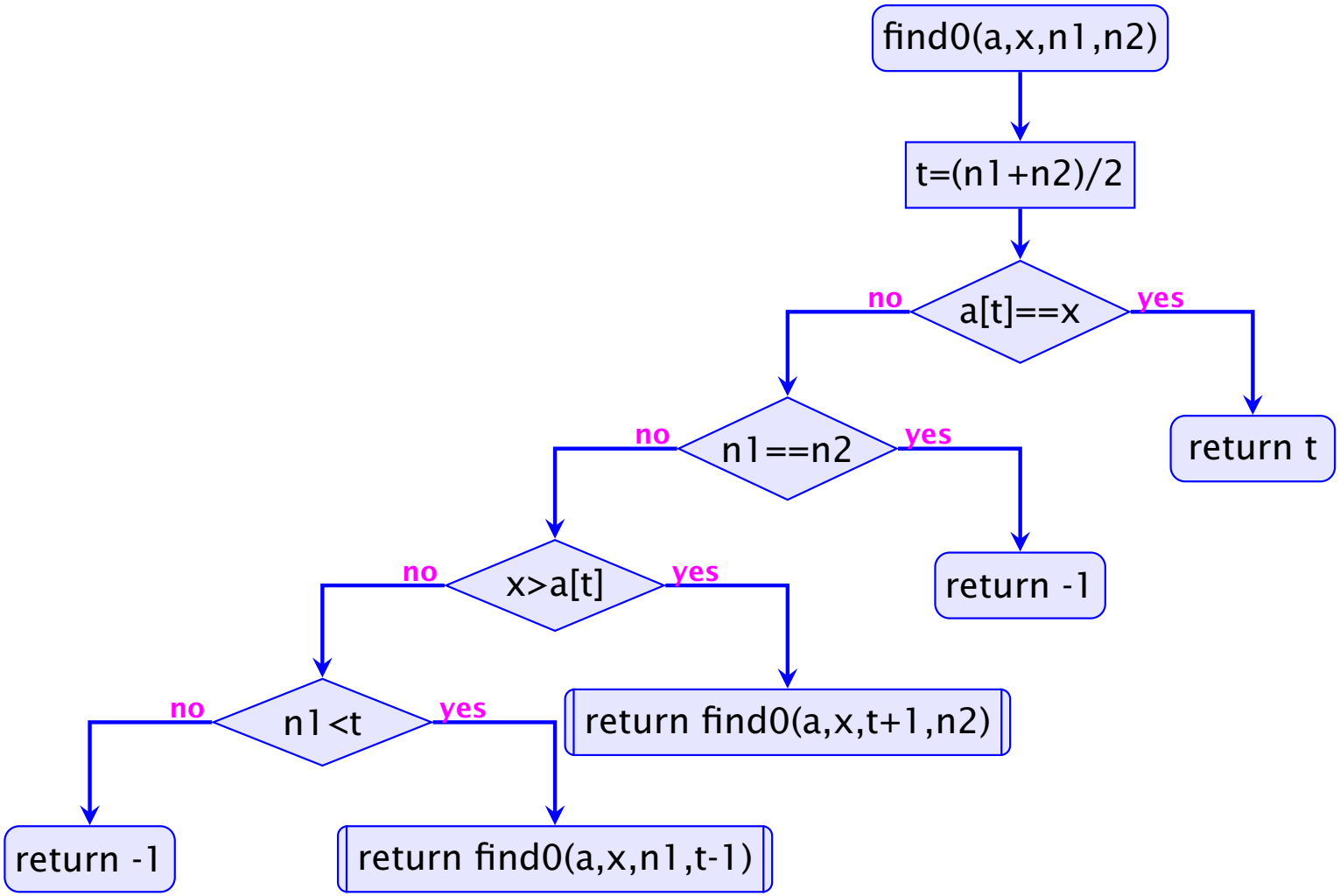
Damit:

```
public static int find(int[] a, int x) {
    return find0(a,x,0,a.length-1);
}
```

Implementierung

```
1  public static int find0(int[] a, int x, int n1, int n2) {
2      int t = (n1 + n2) / 2;
3      if (a[t] == x)
4          return t;
5      else if (n1 == n2)
6          return -1;
7      else if (x > a[t])
8          return find0(a, x, t+1, n2);
9      else if (n1 < t)
10         return find0(a, x, n1, t-1);
11     else return -1;
12 }
```

Kontrollflussdiagramm für `find0`

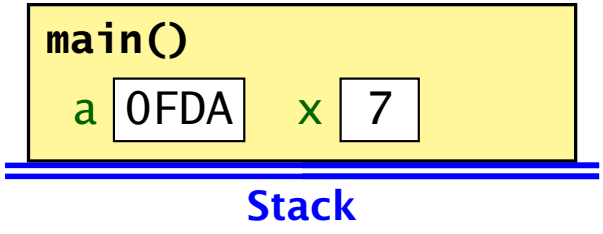


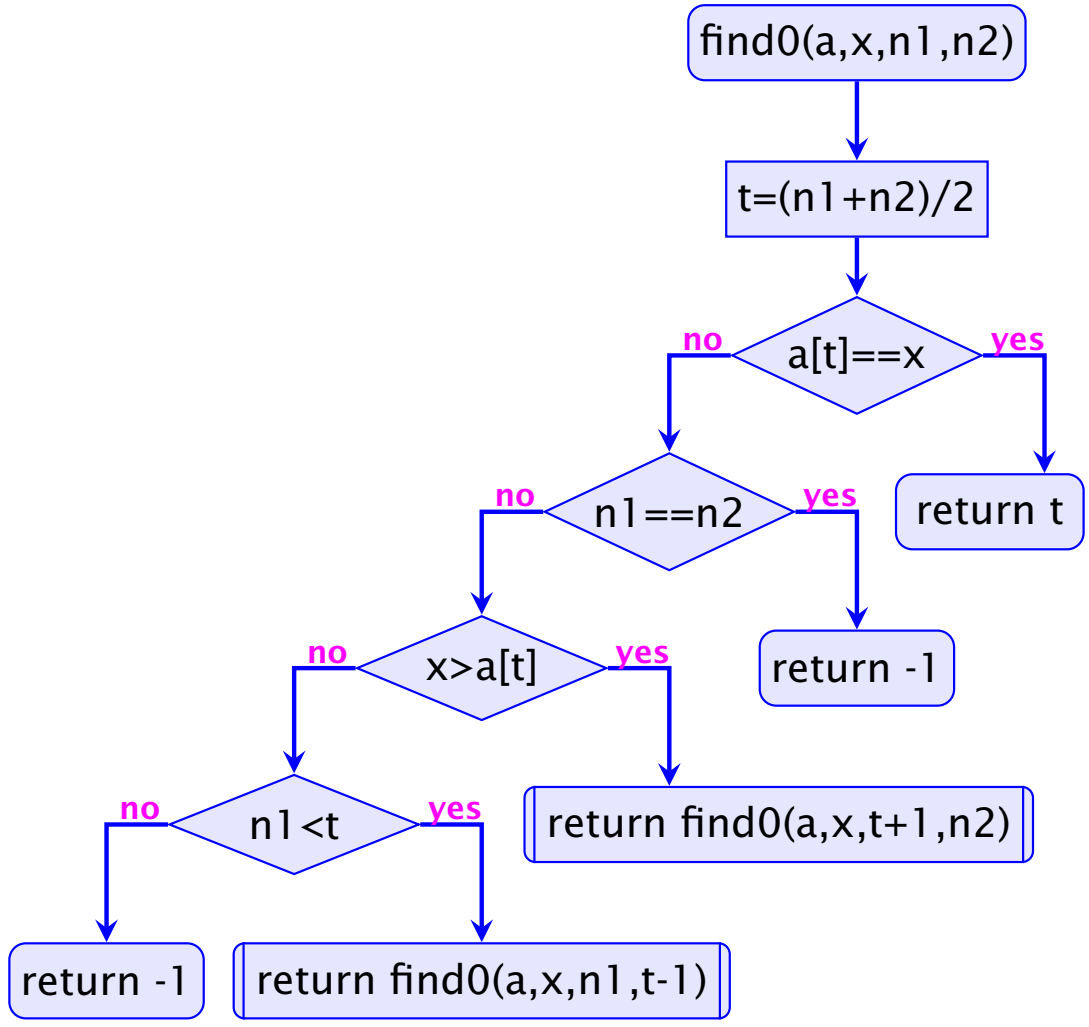
Implementierung

Erläuterungen:

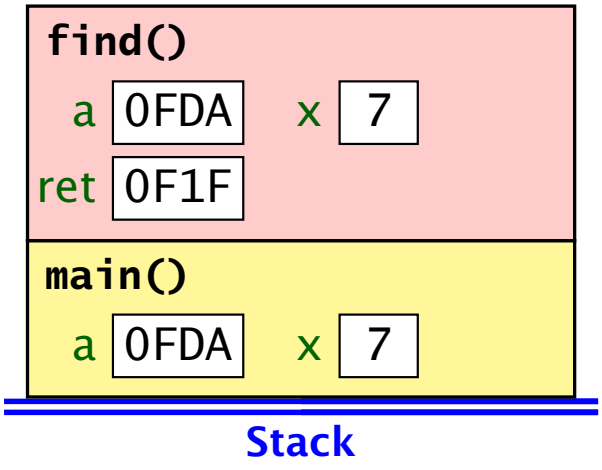
- zwei der `return`-Statements enthalten einen Funktionsaufruf – deshalb die Markierungen an den entsprechenden Knoten.
- (Wir hätten stattdessen auch zwei Knoten und eine Hilfsvariable `result` einführen können)
- `find0()` ruft sich selbst auf.
- Funktionen, die sich selbst (evt. mittelbar) aufrufen, heißen rekursiv.

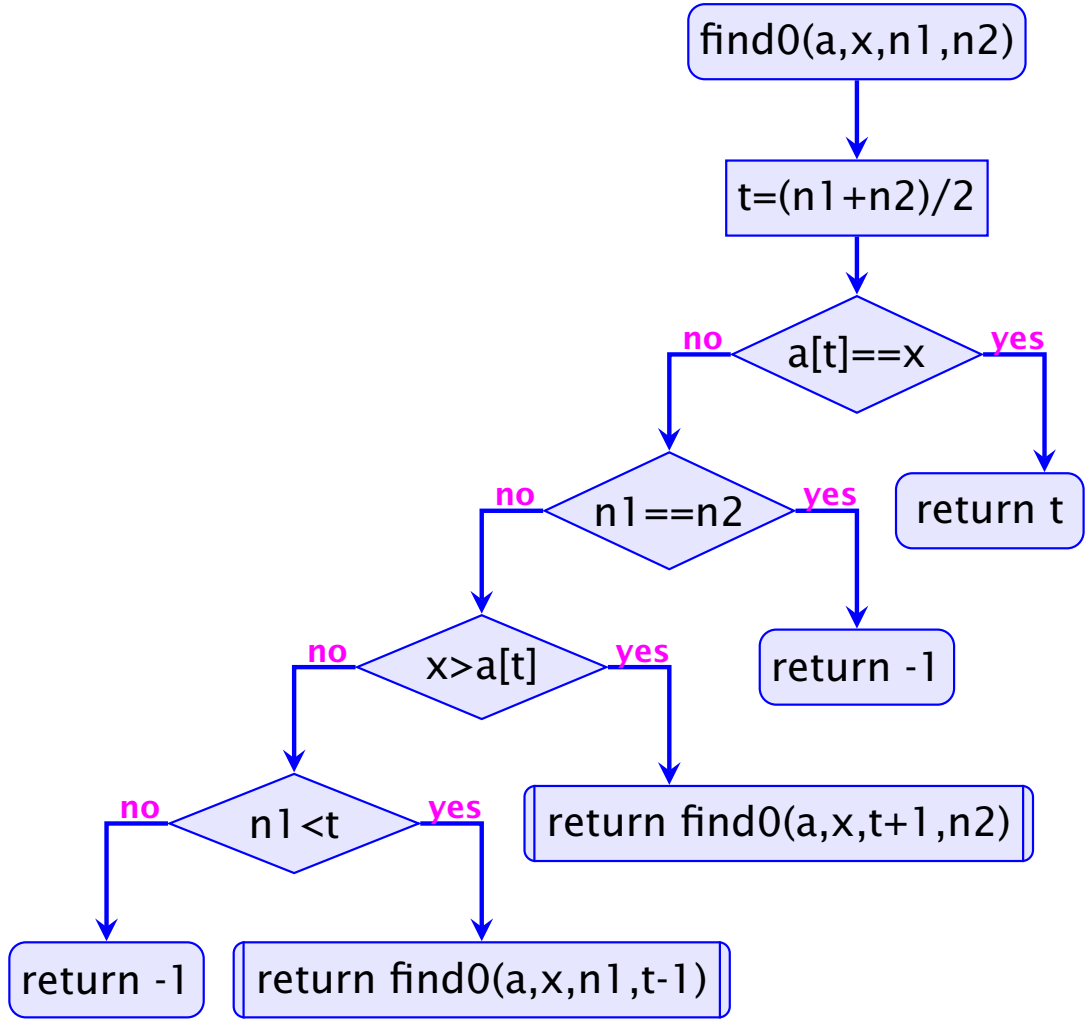
Ausführung


main()
a
0FDA
x
7
Stack


find0(a,x,n1,n2)
t=(n1+n2)/2
a[t]==x
no
yes
return t
n1==n2
no
yes
return -1
x>a[t]
no
yes
return find0(a,x,t+1,n2)
n1<t
no
yes
return -1
return find0(a,x,n1,t-1)

Ausführung


find()
a
0FDA
x
7
ret
0F1F
main()
a
0FDA
x
7
Stack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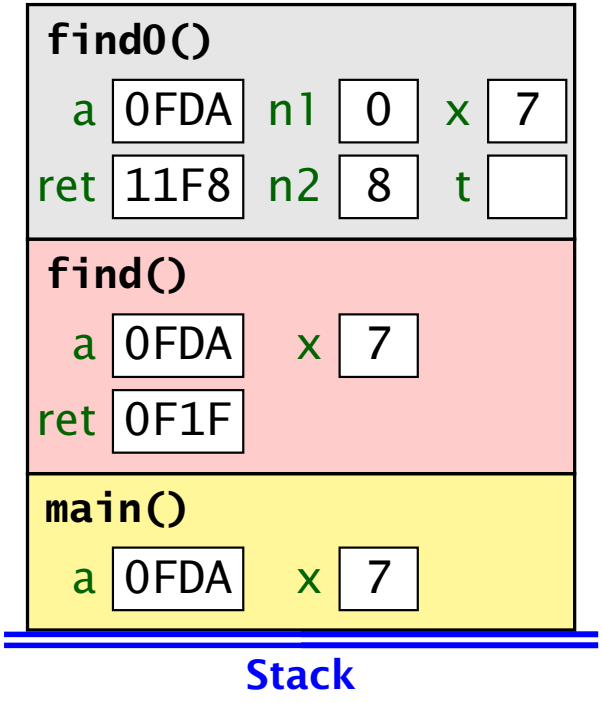
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a 0FDA
n1 0
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ret 11F8
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x 7
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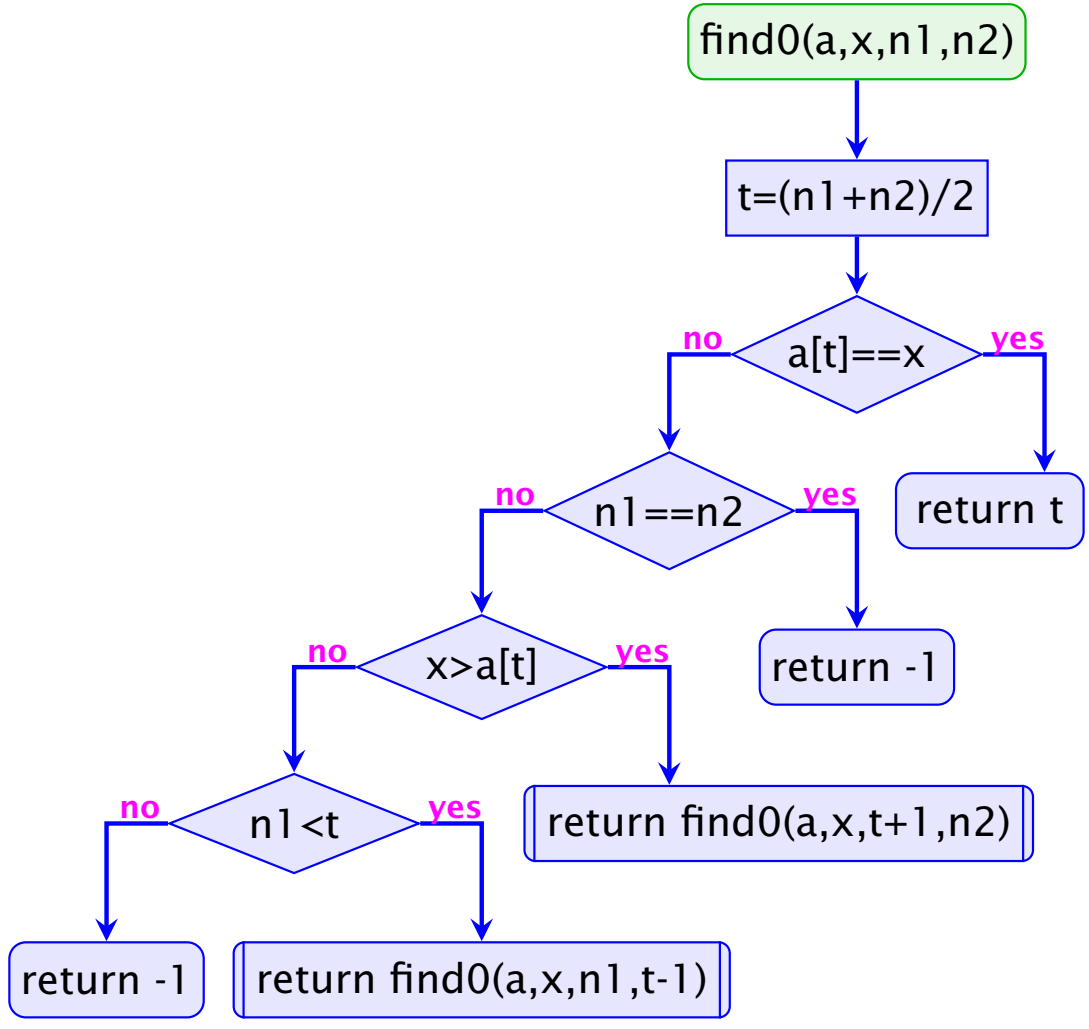
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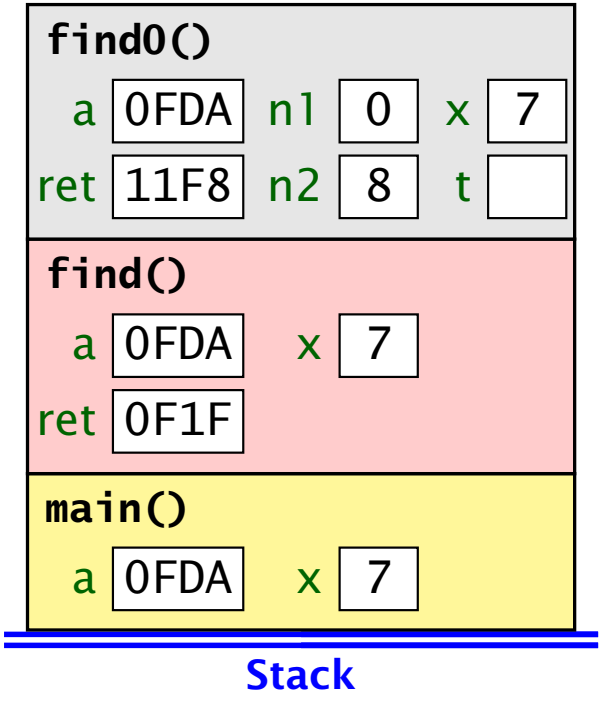
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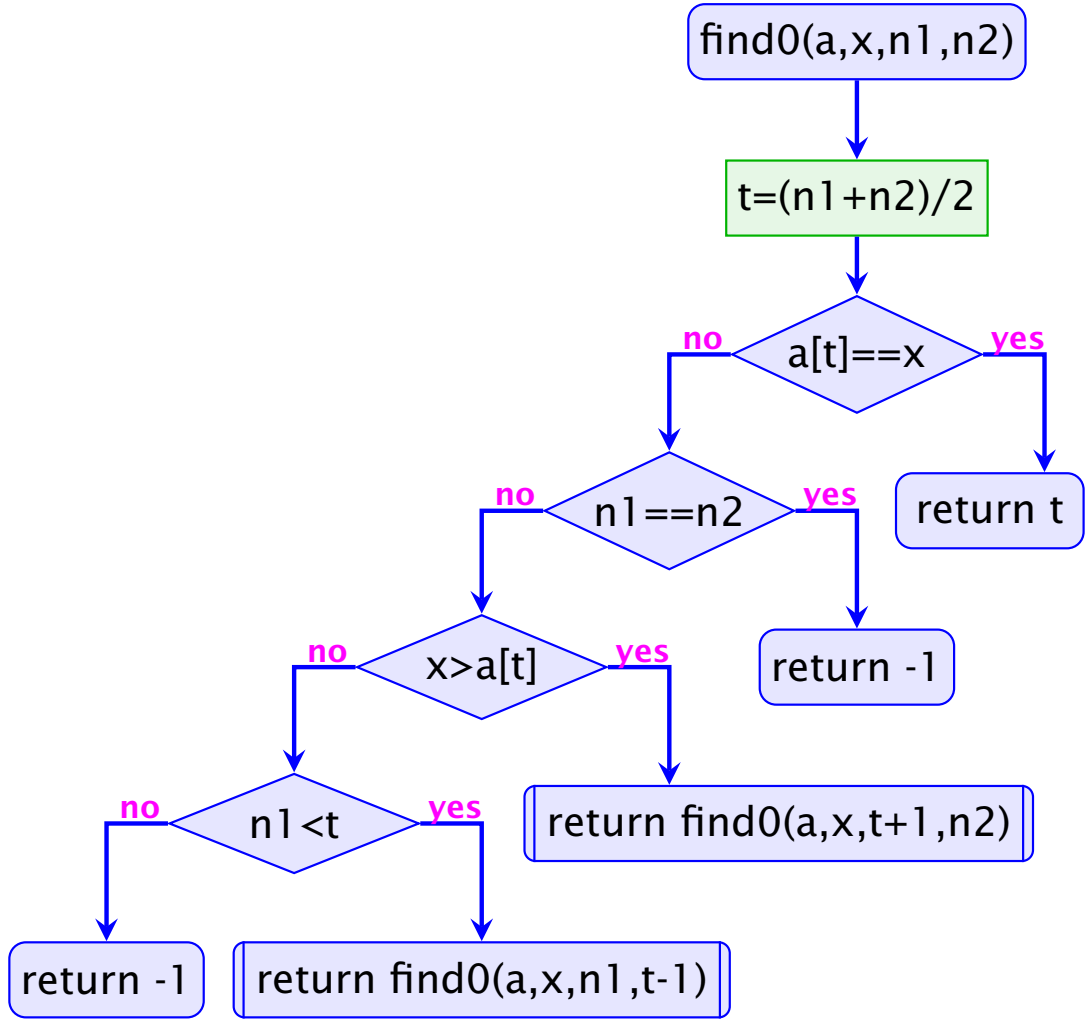
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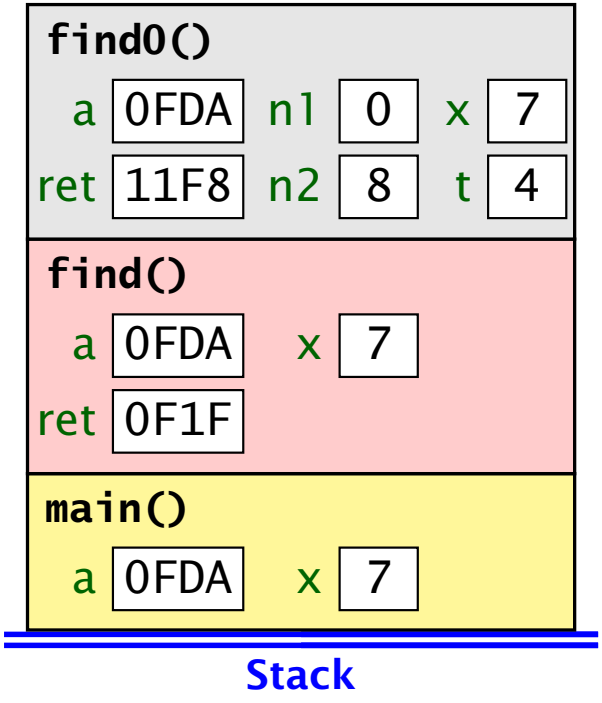
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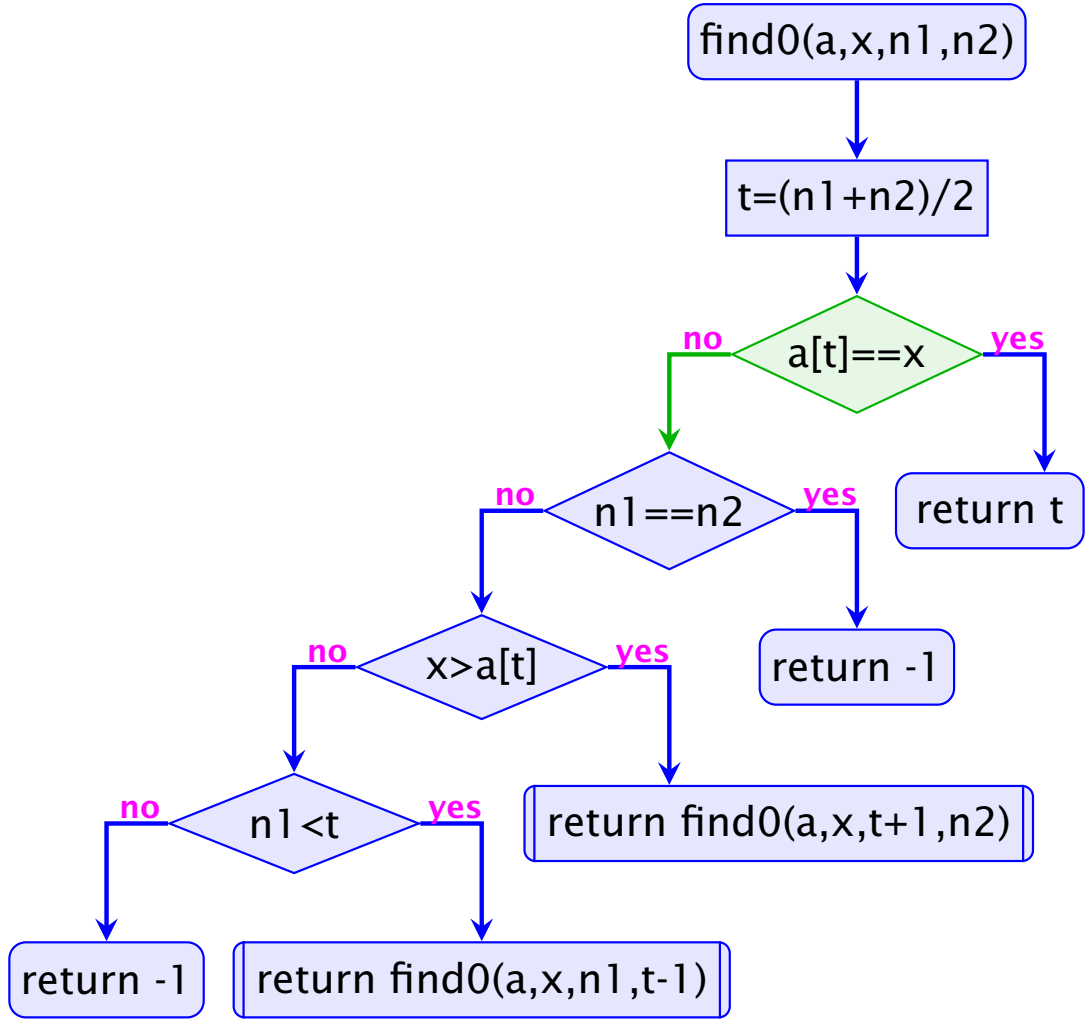
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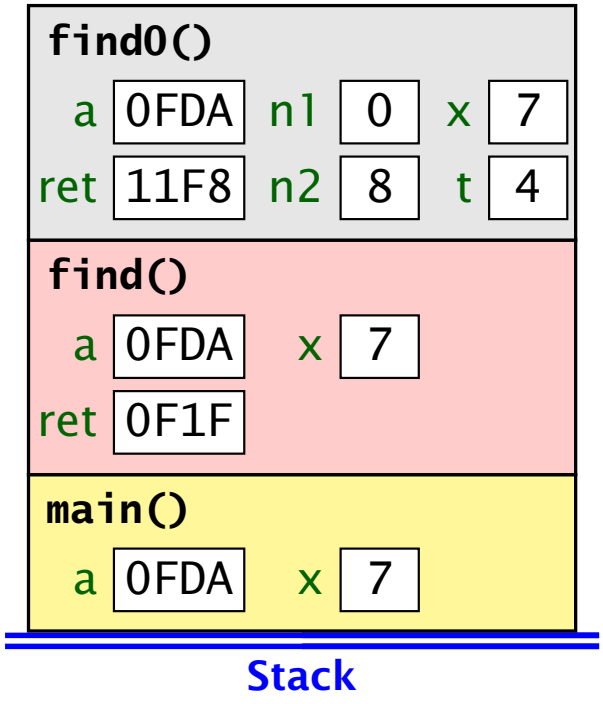
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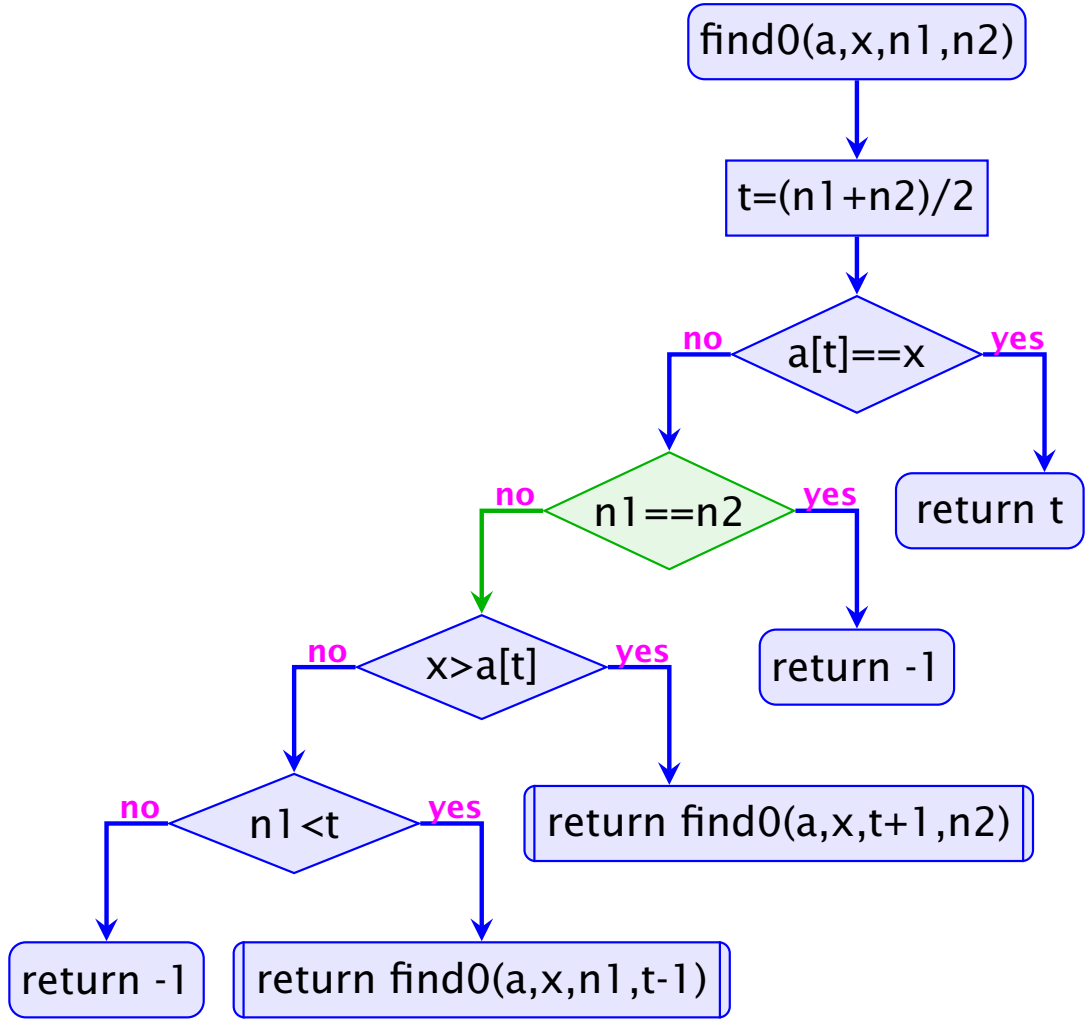
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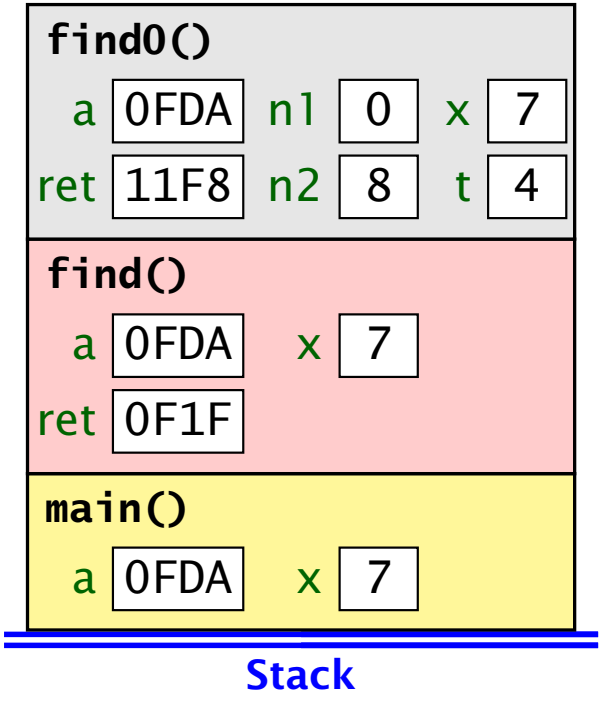
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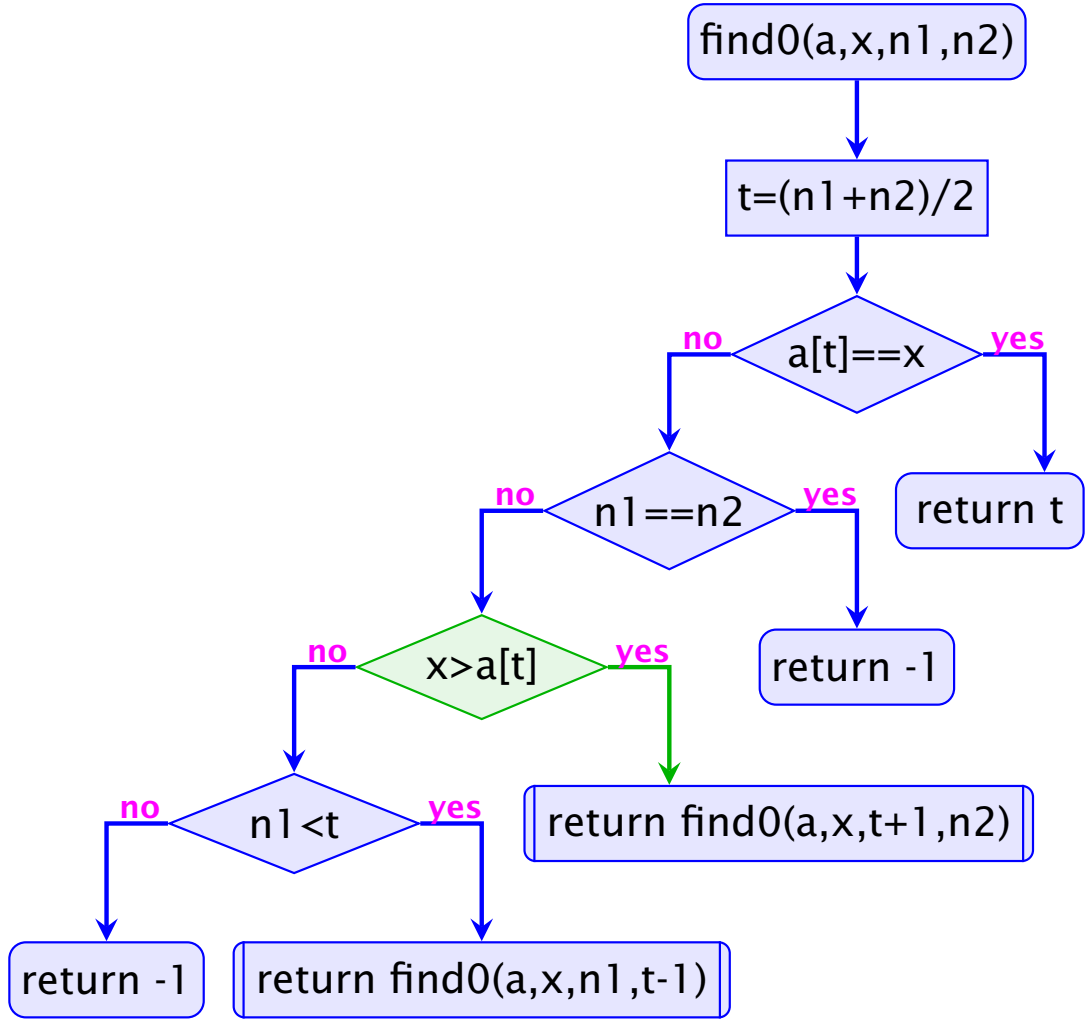
Stack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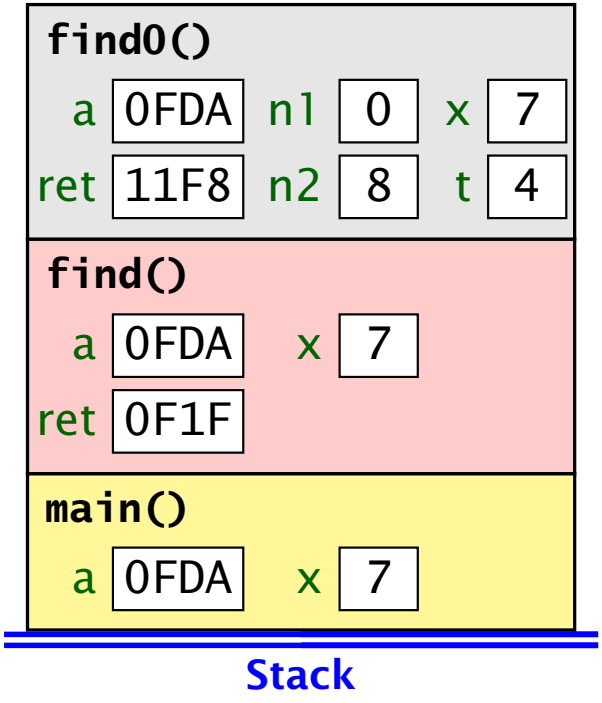
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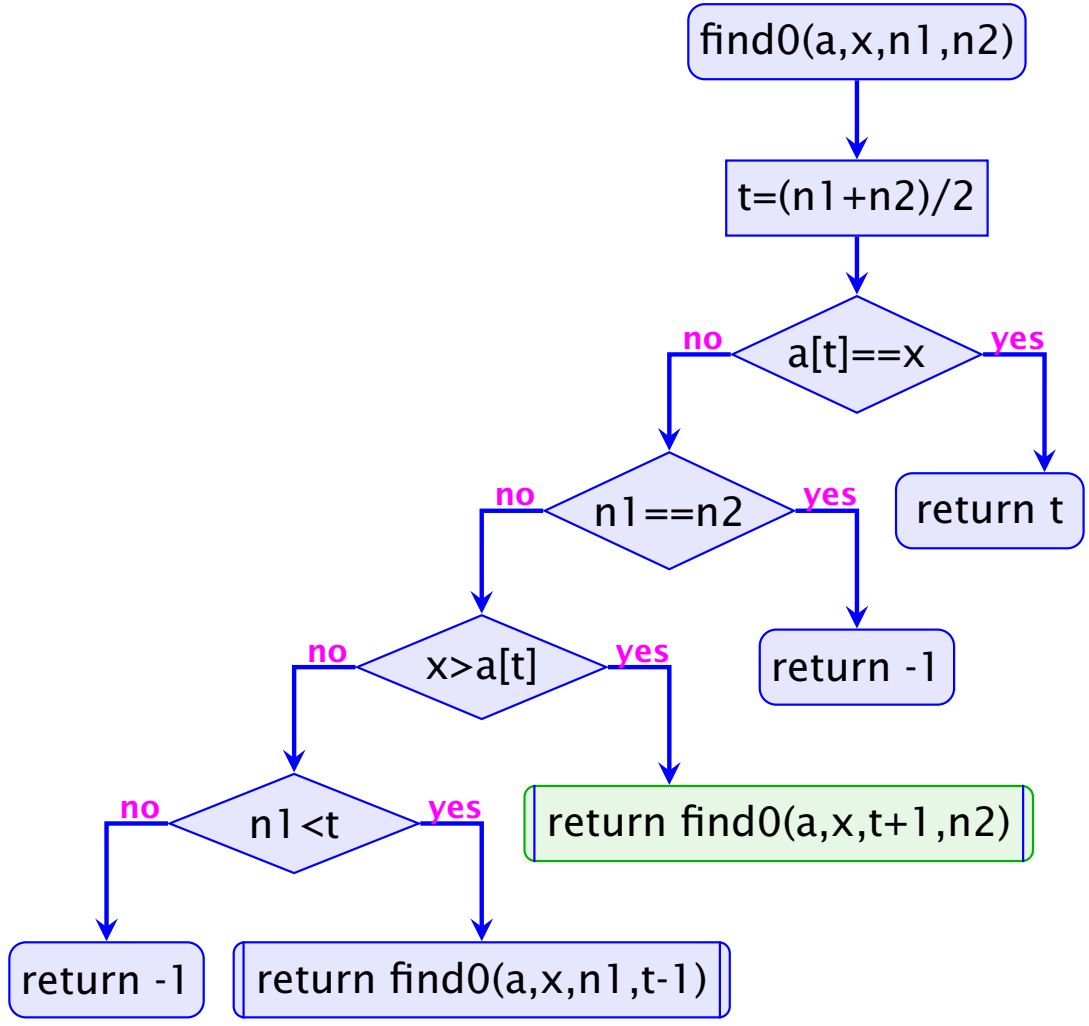
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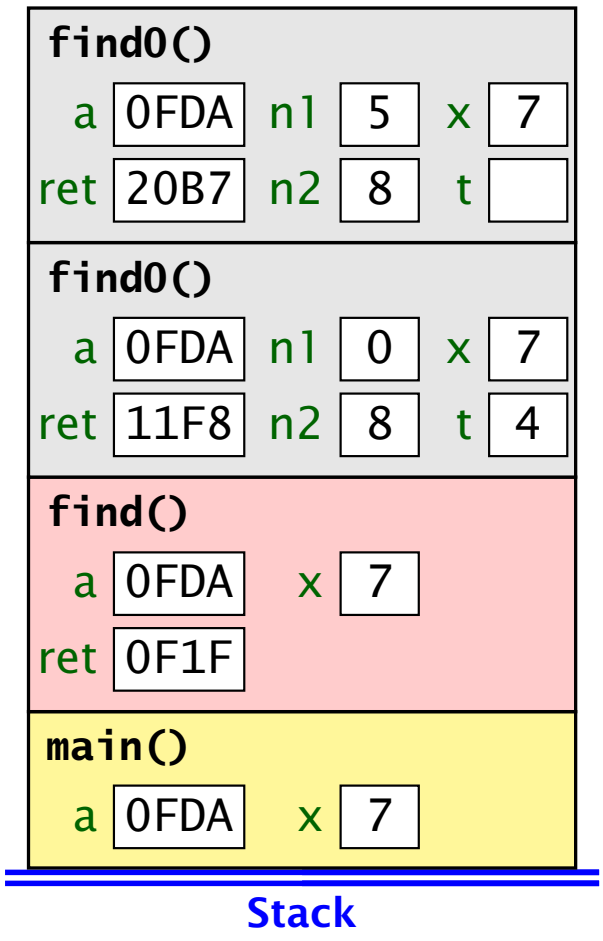
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main()
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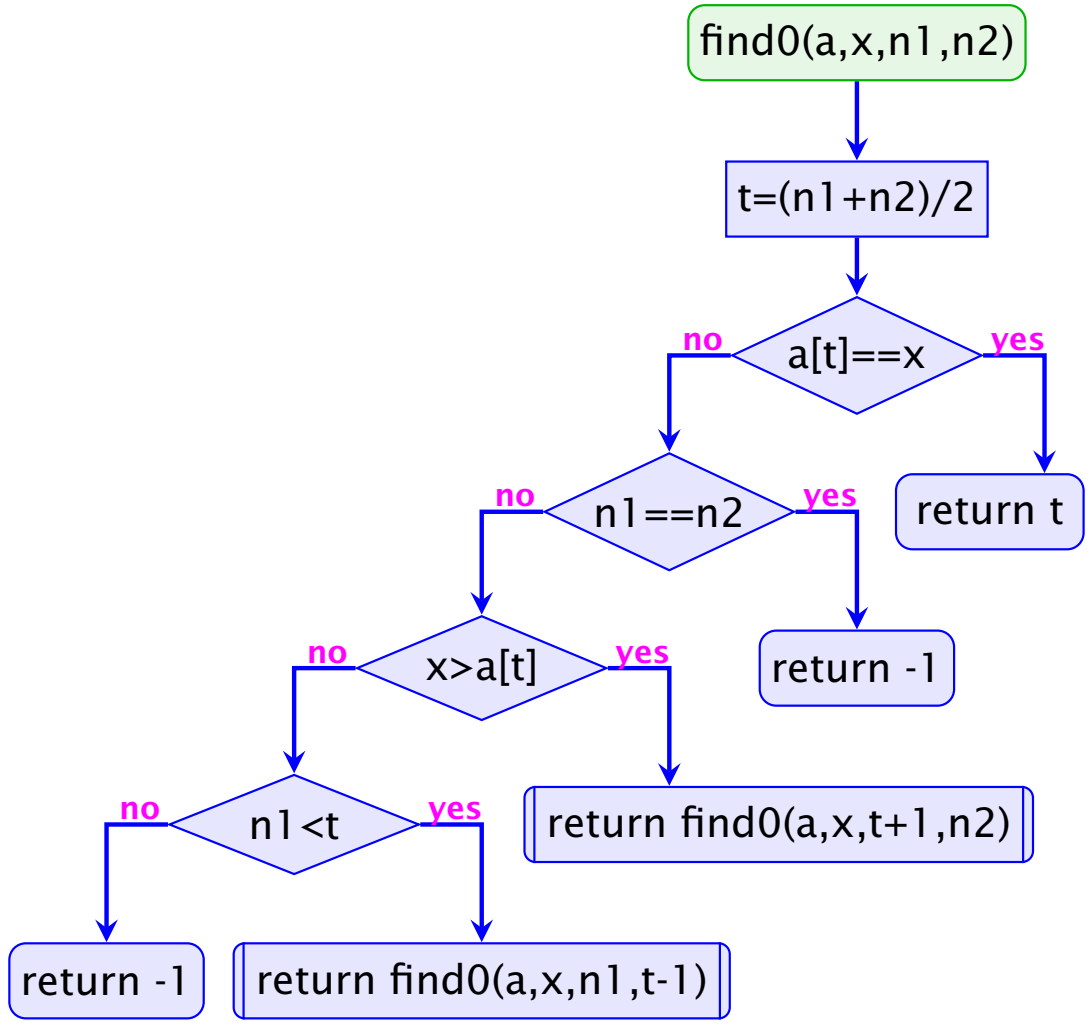
x 7
Stack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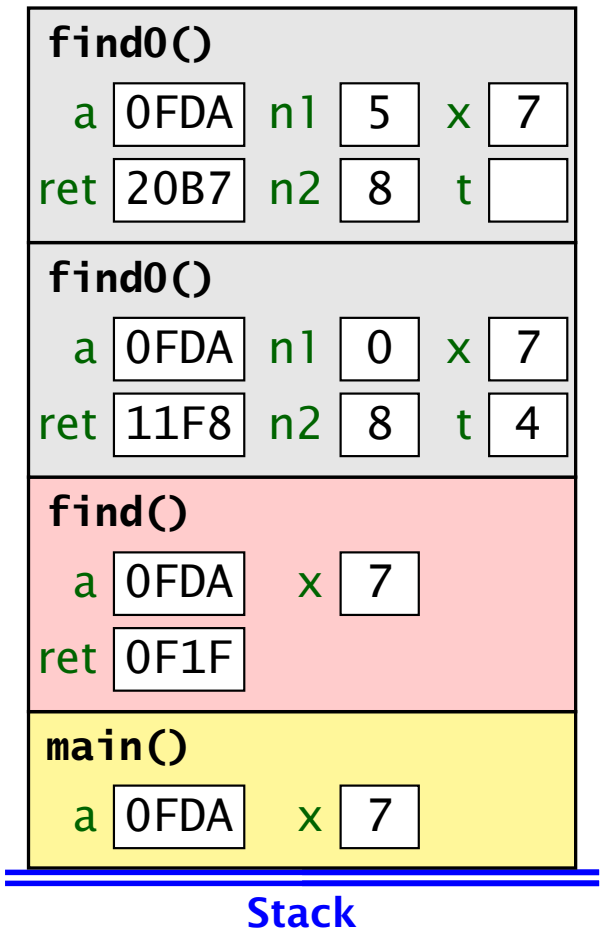
find0()
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n1 5
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ret 20B7
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n1 0
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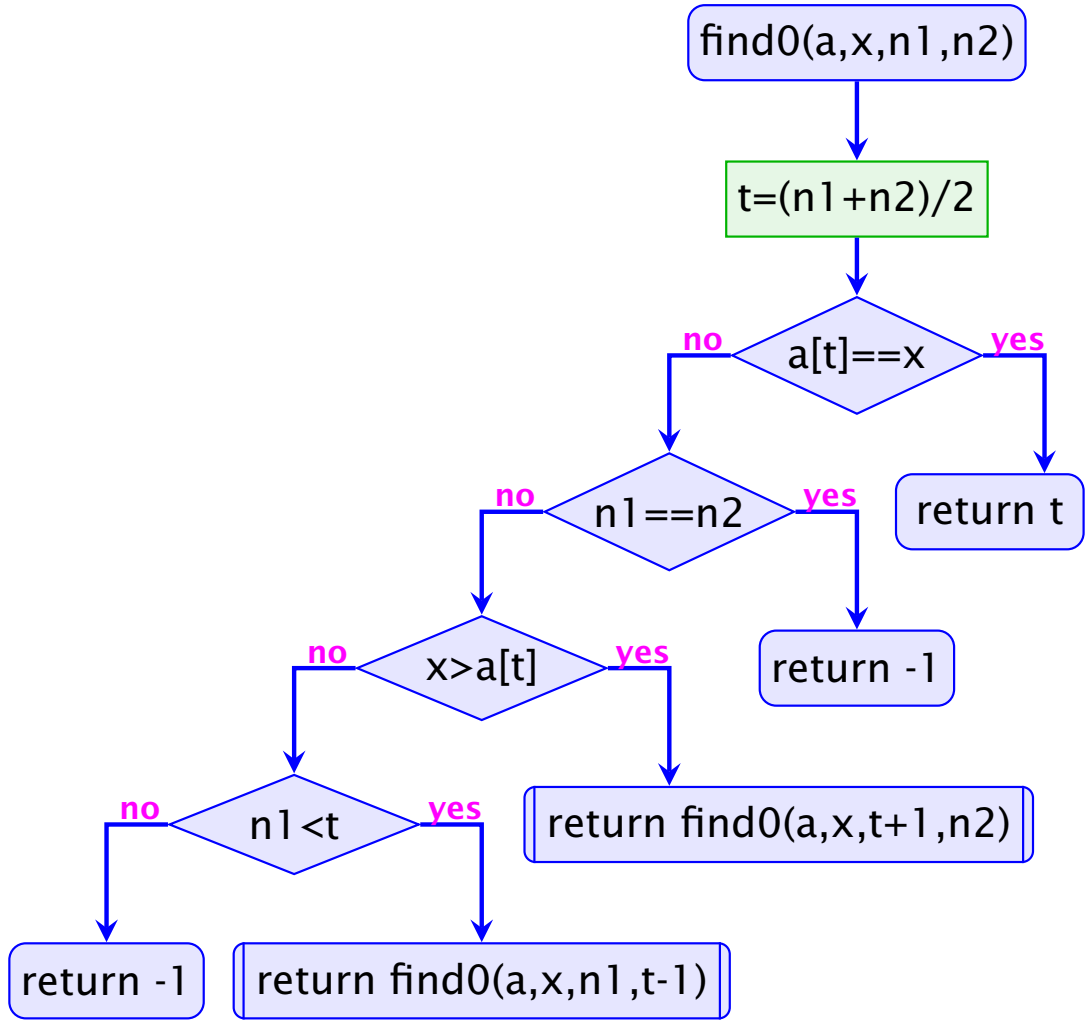
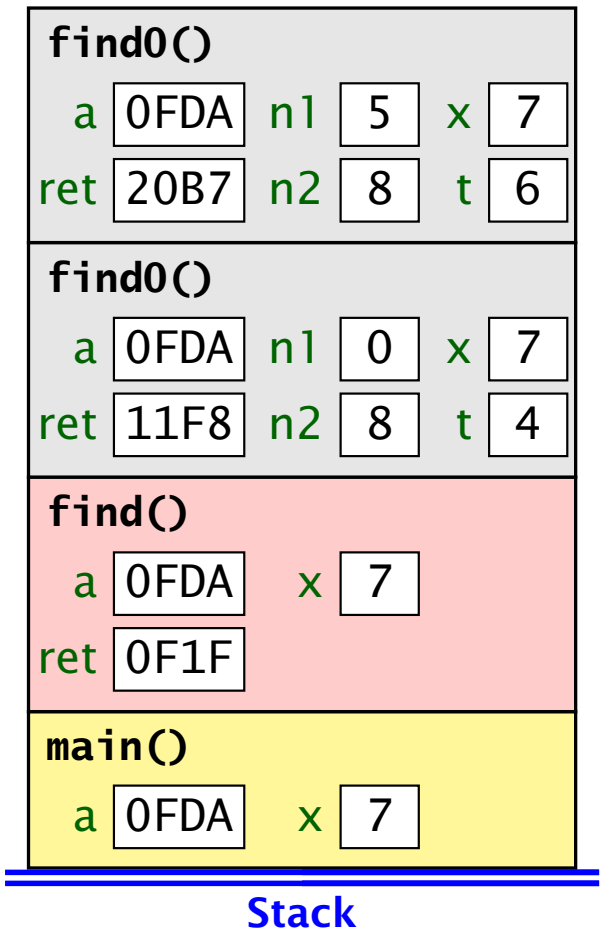
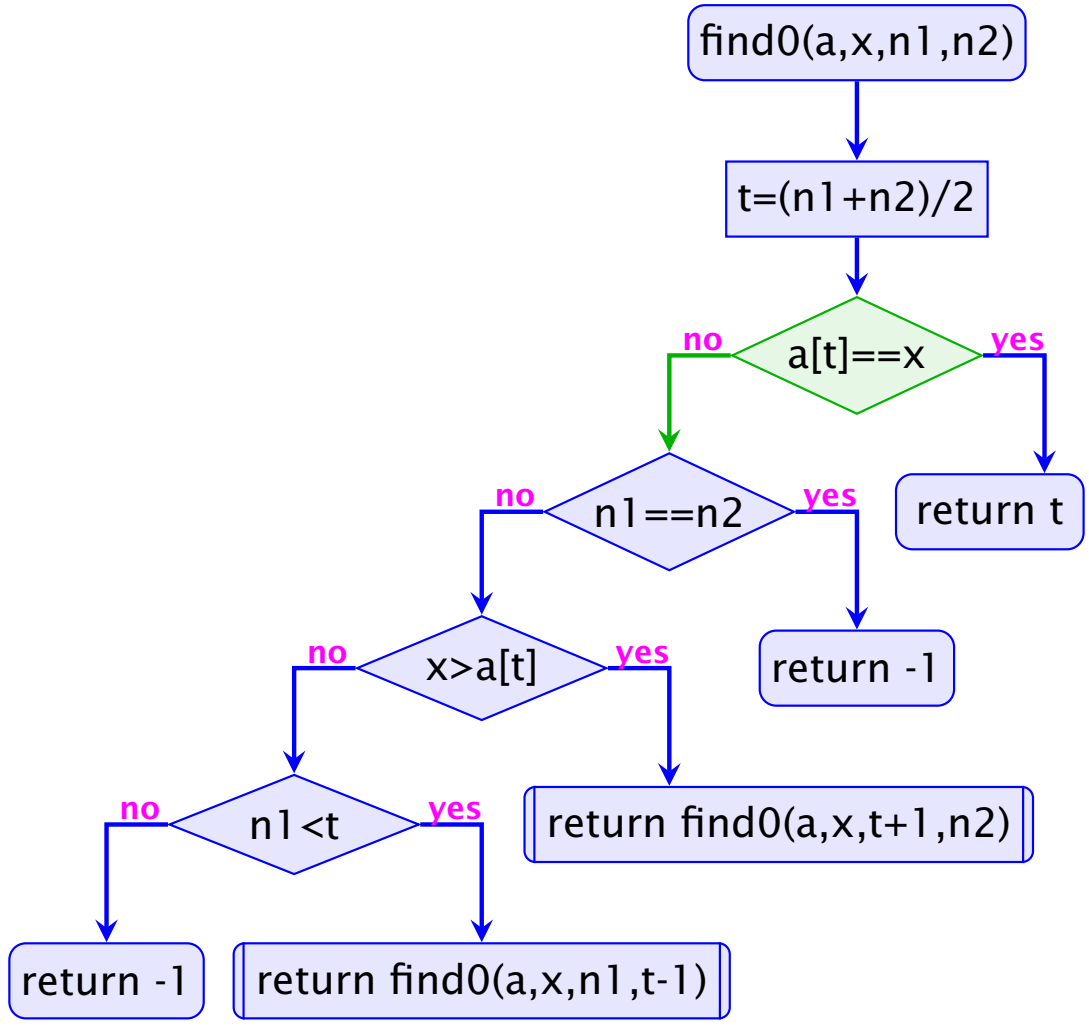
ret 11F8
n2 8
t 4
find()
a 0FDA
x 7
ret 0F1F
main()
a 0FDA
x 7
Stack

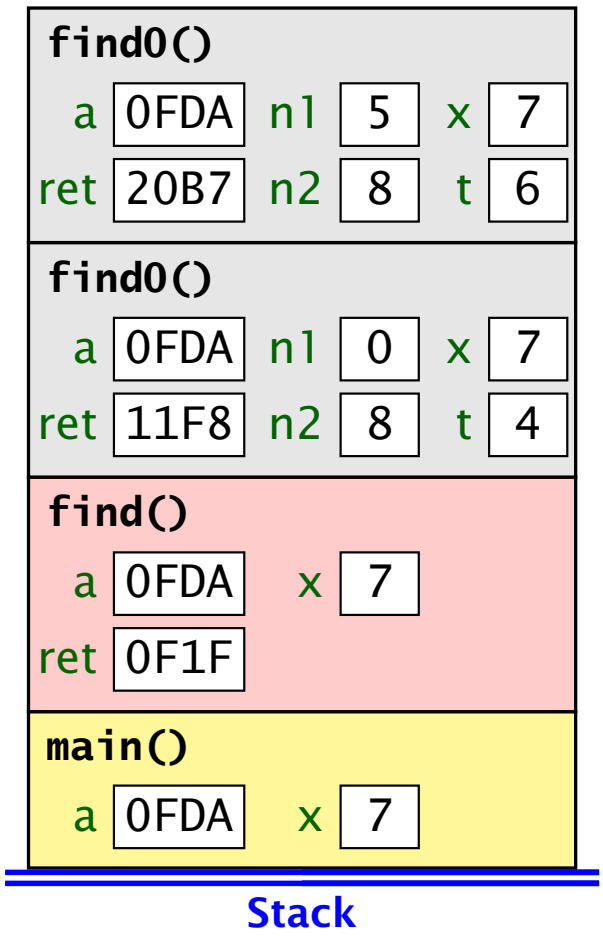

find0(a,x,n1,n2)
t=(n1+n2)/2
a[t]==x
no
yes
return t
n1==n2
no
yes
return -1
x>a[t]
no
yes
return find0(a,x,t+1,n2)
n1<t
no
yes
return -1
return find0(a,x,n1,t-1)

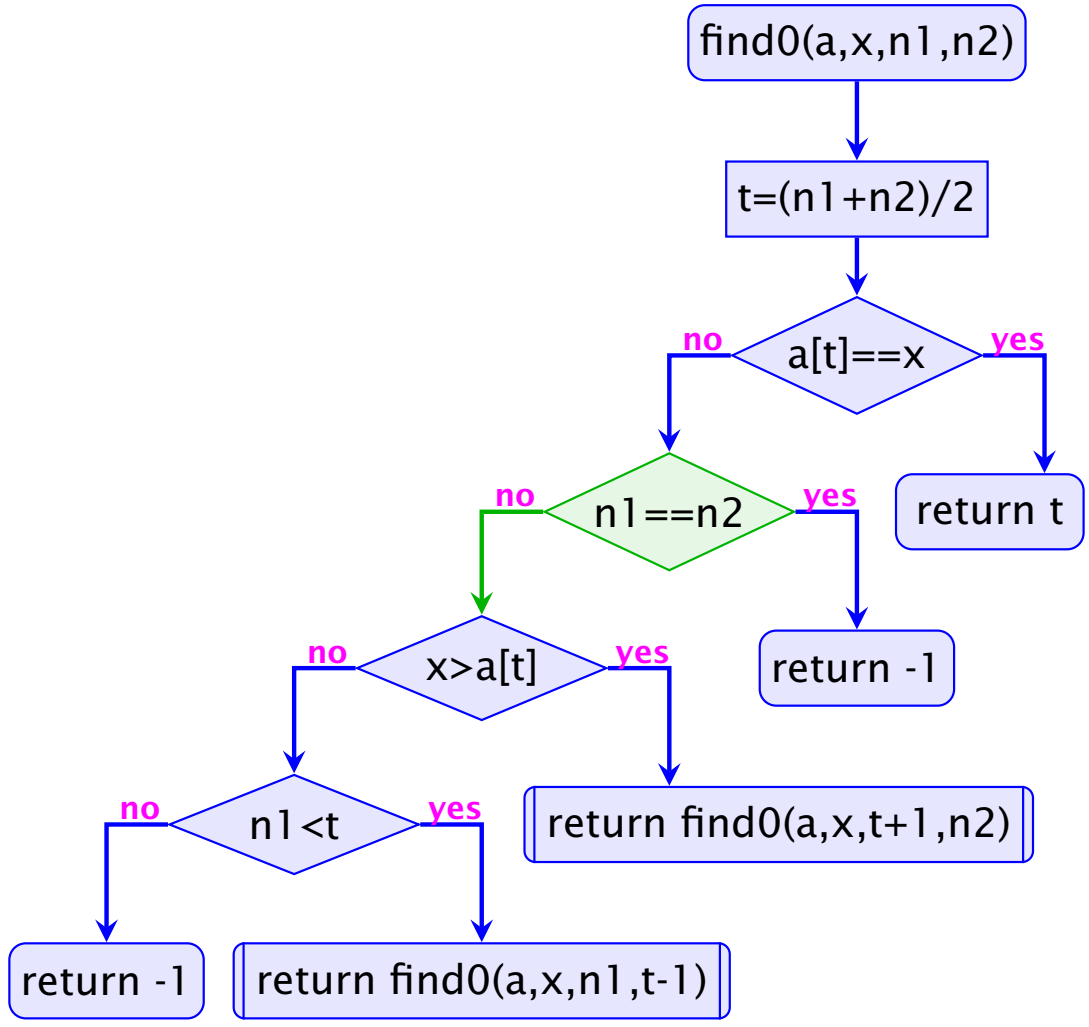
Ausführung


find0()
a 0FDA
n1 5
x 7
ret 20B7
n2 8
t 6
find0()
a 0FDA
n1 0
x 7
ret 11F8
n2 8
t 4
find()
a 0FDA
x 7
ret 0F1F
main()
a 0FDA
x 7
Stack


find0(a,x,n1,n2)
t=(n1+n2)/2
a[t]==x
no
yes
return t
n1==n2
no
yes
return -1
x>a[t]
no
yes
return find0(a,x,t+1,n2)
n1<t
no
yes
return -1
return find0(a,x,n1,t-1)

Ausführung


find0()
a 0FDA
n1 5
x 7
ret 20B7
n2 8
t 6
find0()
a 0FDA
n1 0
x 7
ret 11F8
n2 8
t 4
find()
a 0FDA
x 7
ret 0F1F
main()
a 0FDA
x 7
Stack


find0(a,x,n1,n2)
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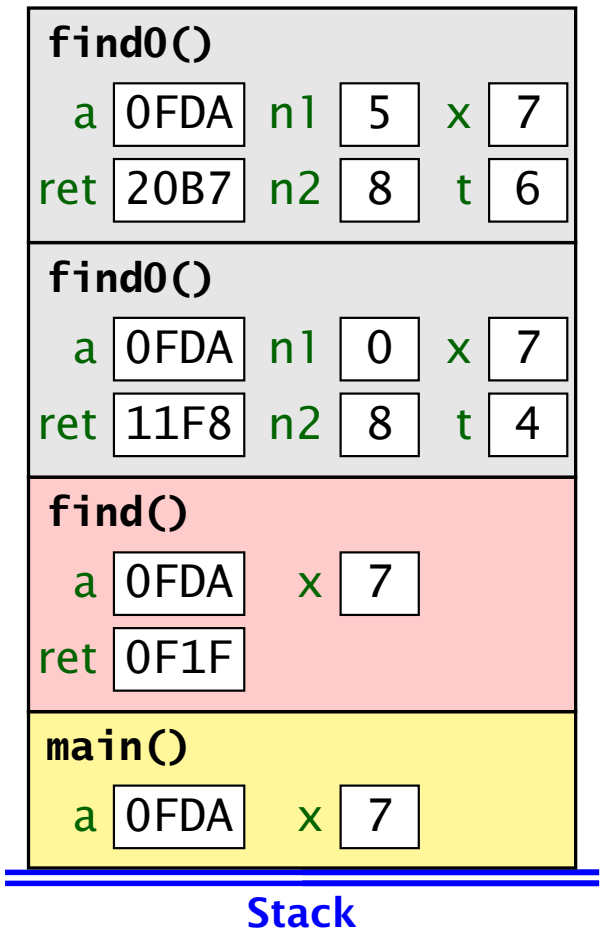
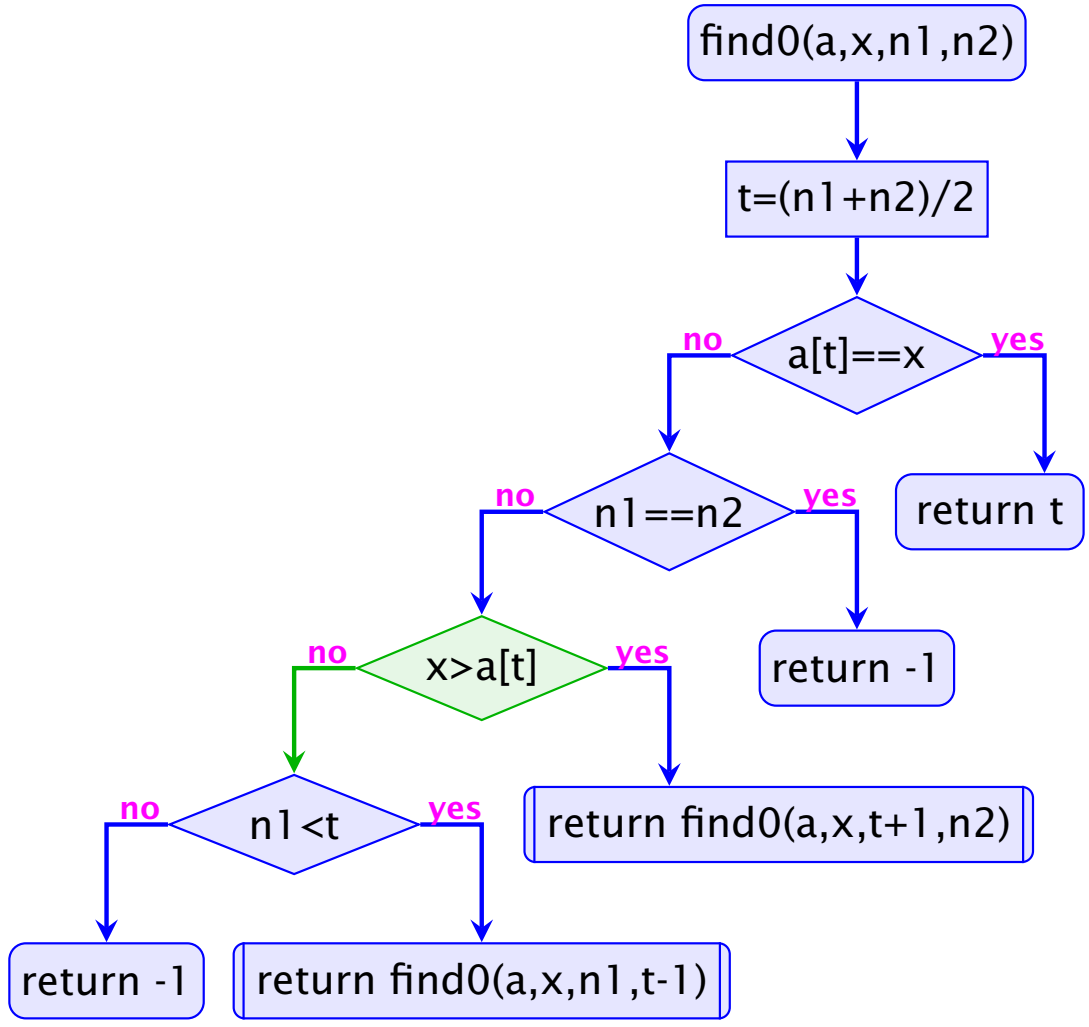
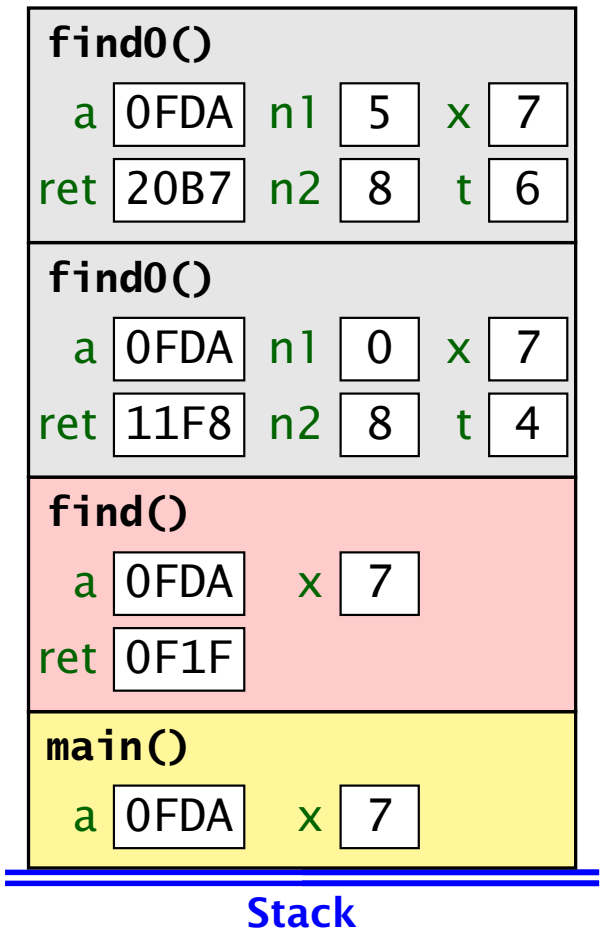
a[t]==x
no
yes
return t
n1==n2
no
yes
return -1
x>a[t]
no
yes
return find0(a,x,t+1,n2)
n1<t
no
yes
return -1
return find0(a,x,n1,t-1)

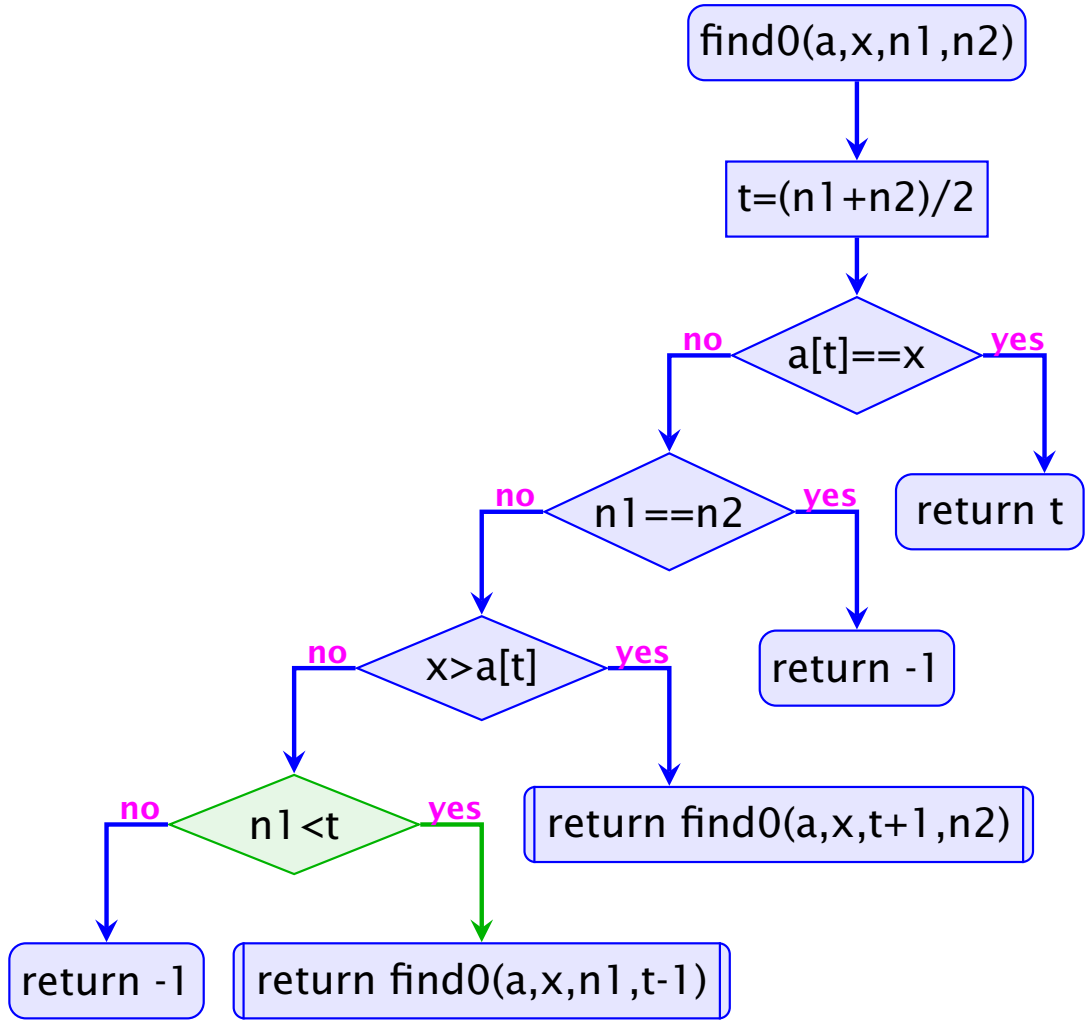
Ausführung


find0()
a 0FDA
n1 5
x 7
ret 20B7
n2 8
t 6
find0()
a 0FDA
n1 0
x 7
ret 11F8
n2 8
t 4
find()
a 0FDA
x 7
ret 0F1F
main()
a 0FDA
x 7
Stack

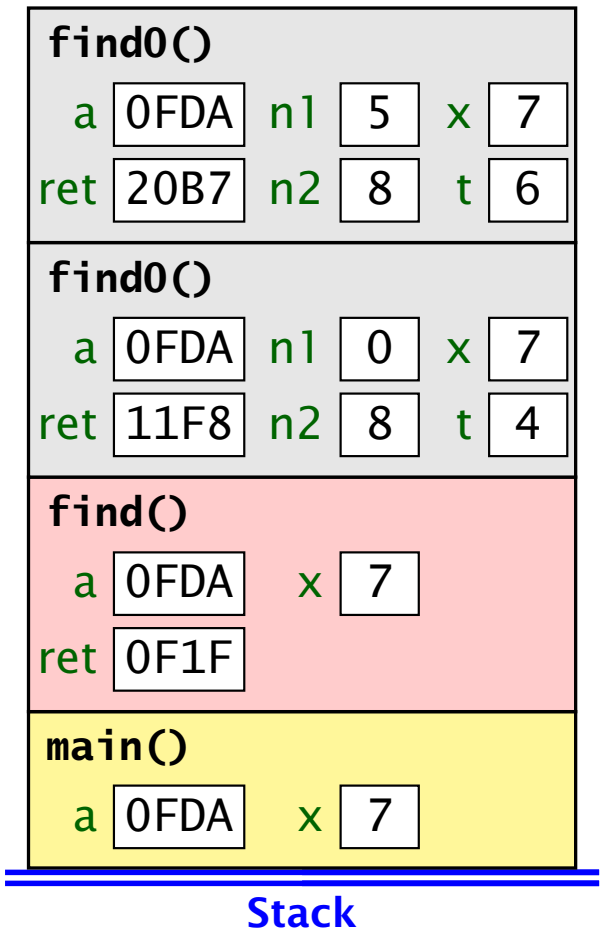

find0(a,x,n1,n2)
t=(n1+n2)/2
a[t]==x
no
yes
return t
n1==n2
no
yes
return -1
x>a[t]
no
yes
return find0(a,x,t+1,n2)
n1<t
no
yes
return -1
return find0(a,x,n1,t-1)

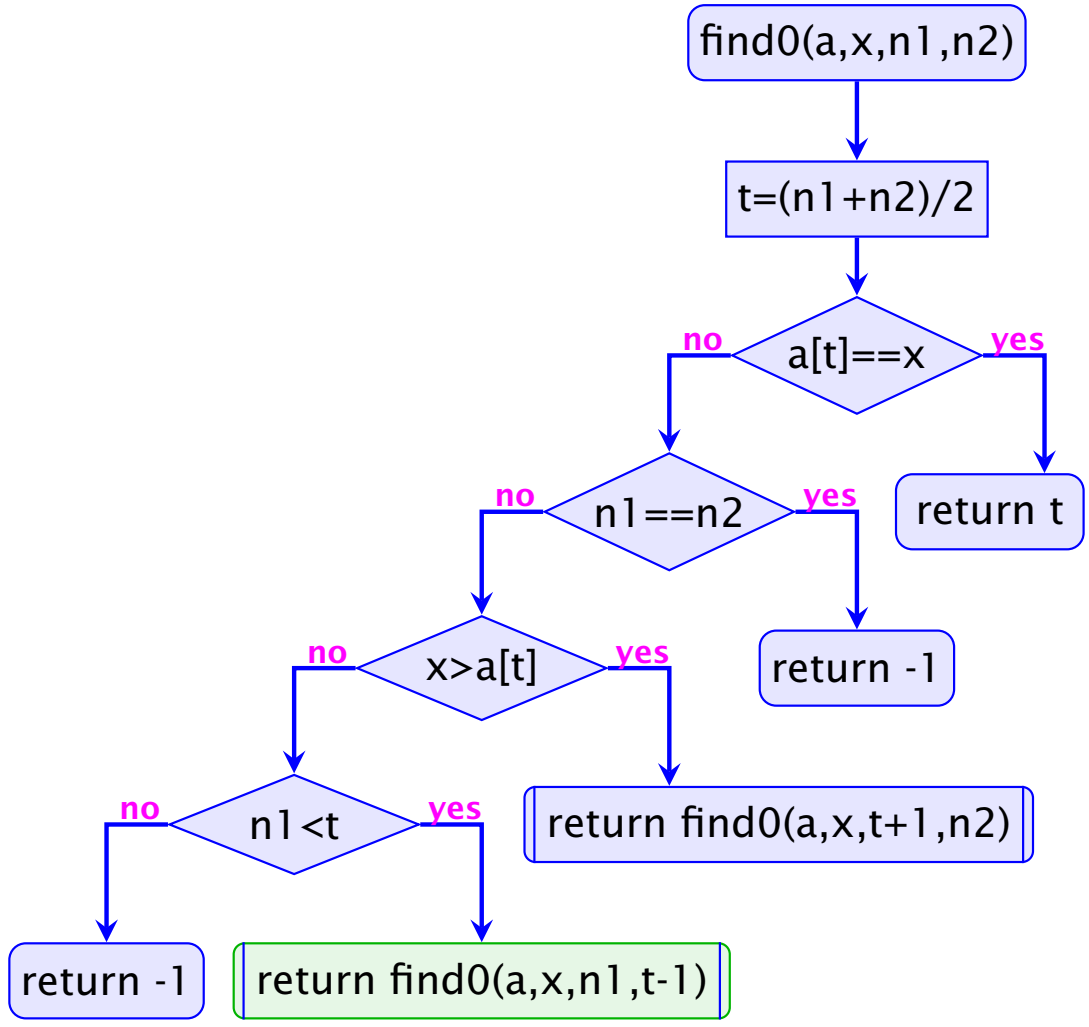
Ausführung


find0()
a 0FDA
n1 5
x 7
ret 20B7
n2 8
t 6
find0()
a 0FDA
n1 0
x 7
ret 11F8
n2 8
t 4
find()
a 0FDA
x 7
ret 0F1F
main()
a 0FDA
x 7
Stack

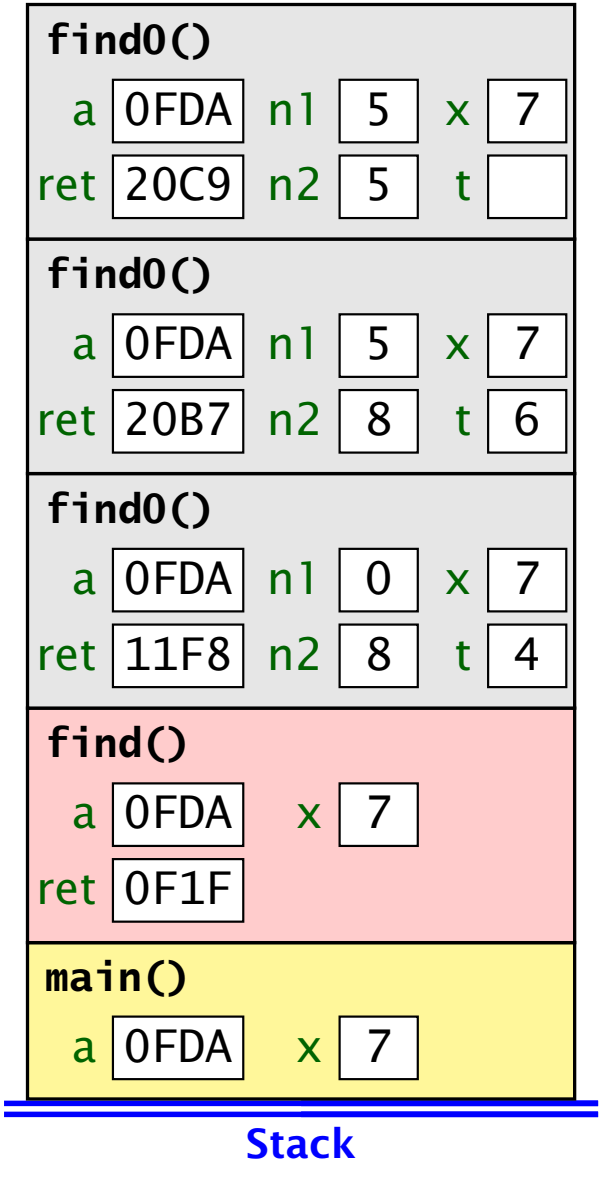

find0(a,x,n1,n2)
t=(n1+n2)/2
a[t]==x
no
yes
return t
n1==n2
no
yes
return -1
x>a[t]
no
yes
return find0(a,x,t+1,n2)
n1<t
no
yes
return -1
return find0(a,x,n1,t-1)

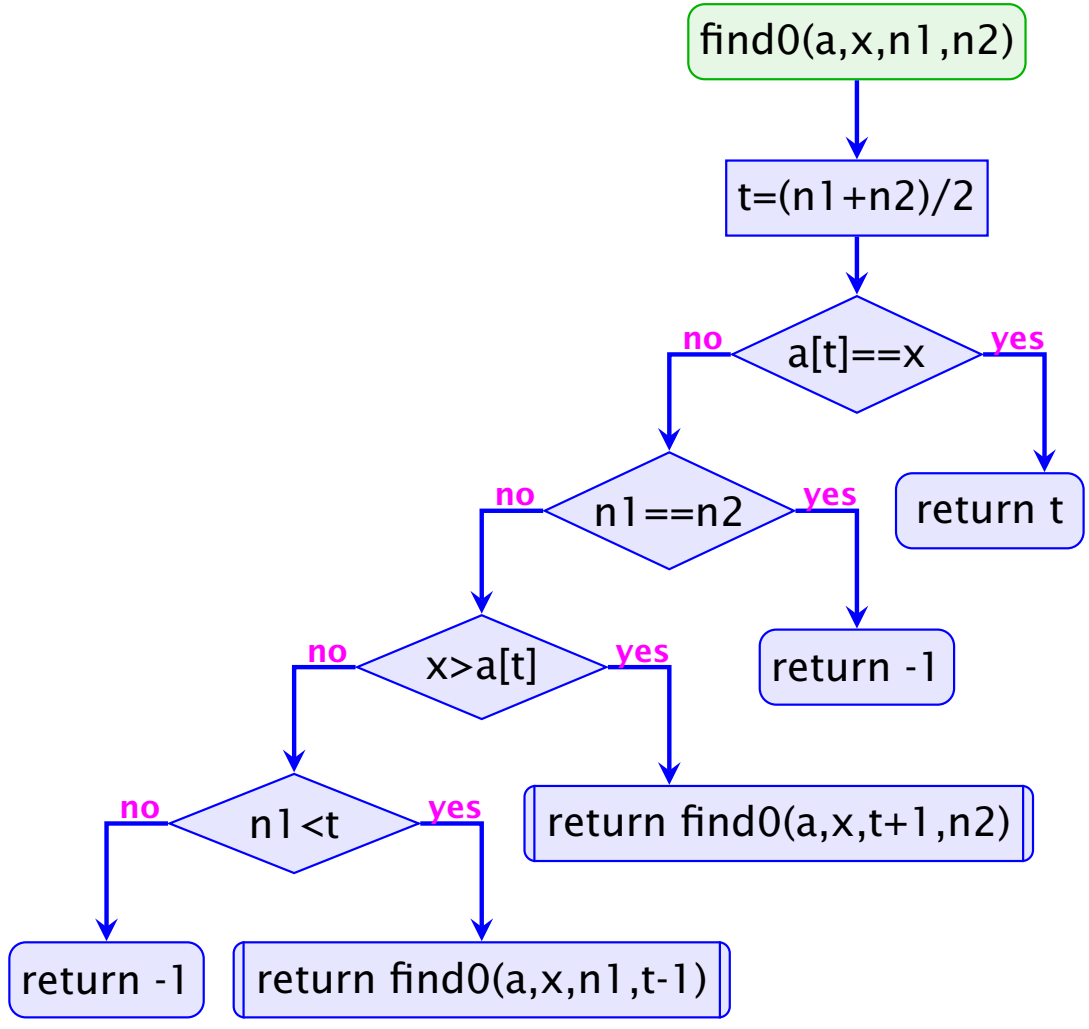
Ausführung


find0()
a 0FDA
n1 5
x 7
ret 20B7
n2 8
t 6
find0()
a 0FDA
n1 0
x 7
ret 11F8
n2 8
t 4
find()
a 0FDA
x 7
ret 0F1F
main()
a 0FDA
x 7
Stack

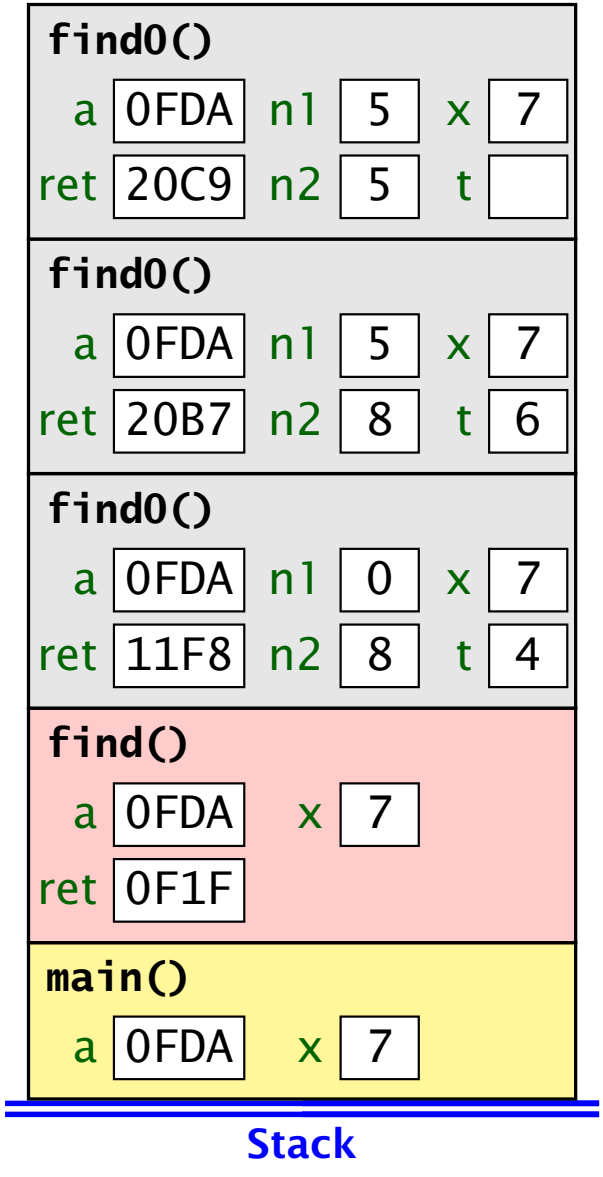

find0(a,x,n1,n2)
t=(n1+n2)/2
a[t]==x
no
yes
return t
n1==n2
no
yes
return -1
x>a[t]
no
yes
return find0(a,x,t+1,n2)
n1<t
no
yes
return -1
return find0(a,x,n1,t-1)

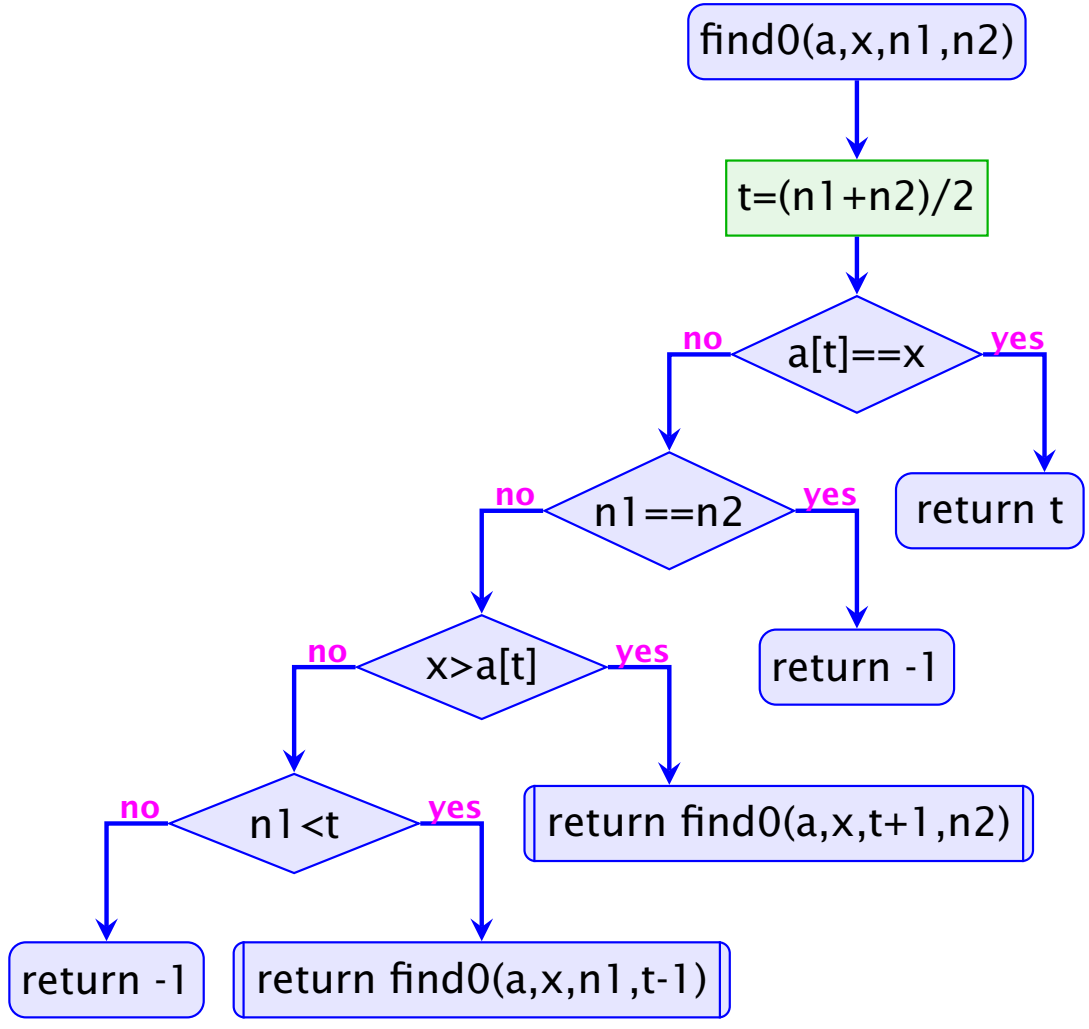
Ausführung


find0()
a 0FDA n1 5 x 7
ret 20C9 n2 5 t
find0()
a 0FDA n1 5 x 7
ret 20B7 n2 8 t 6
find0()
a 0FDA n1 0 x 7
ret 11F8 n2 8 t 4
find()
a 0FDA x 7
ret 0F1F
main()
a 0FDA x 7
Stack

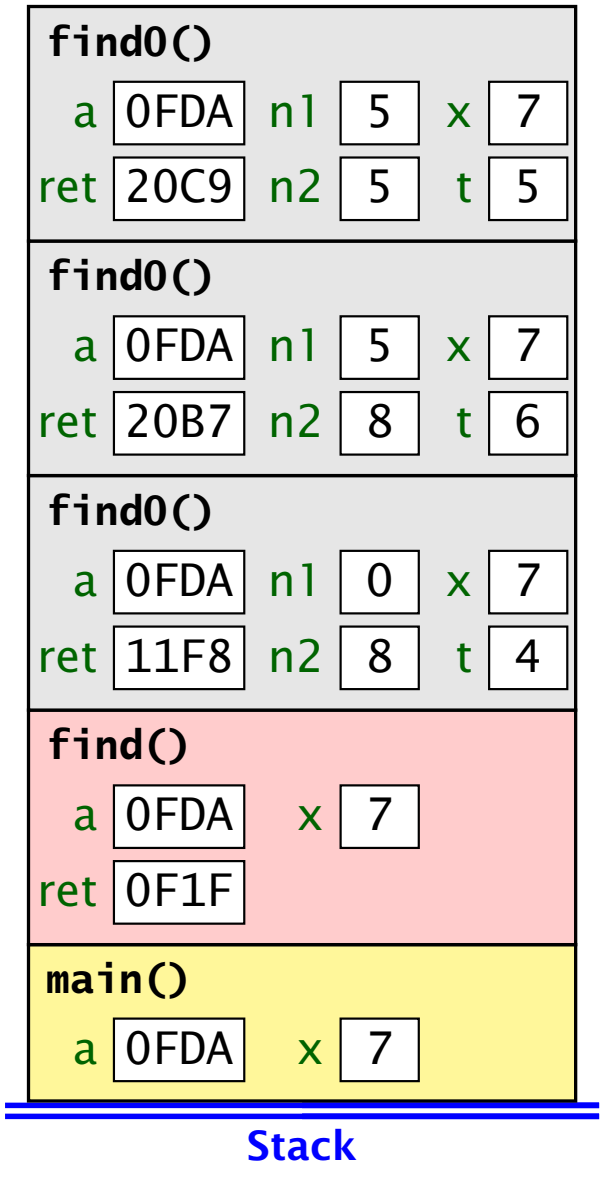

find0(a,x,n1,n2)
t=(n1+n2)/2
a[t]==x
no
yes
return t
n1==n2
no
yes
return -1
x>a[t]
no
yes
return find0(a,x,t+1,n2)
n1<t
no
yes
return -1
return find0(a,x,n1,t-1)

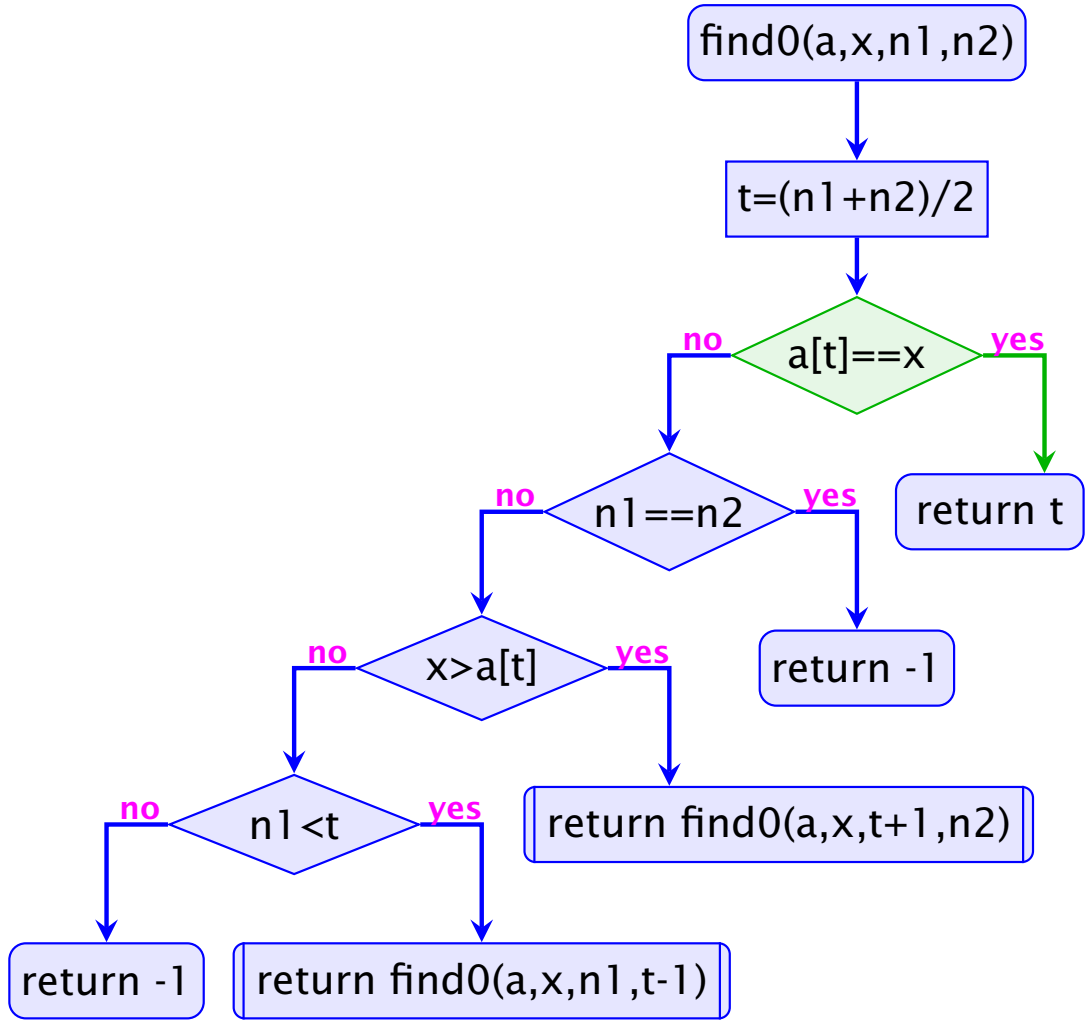
Ausführung


find0()
a 0FDA n1 5 x 7
ret 20C9 n2 5 t
find0()
a 0FDA n1 5 x 7
ret 20B7 n2 8 t 6
find0()
a 0FDA n1 0 x 7
ret 11F8 n2 8 t 4
find()
a 0FDA x 7
ret 0F1F
main()
a 0FDA x 7
Stack

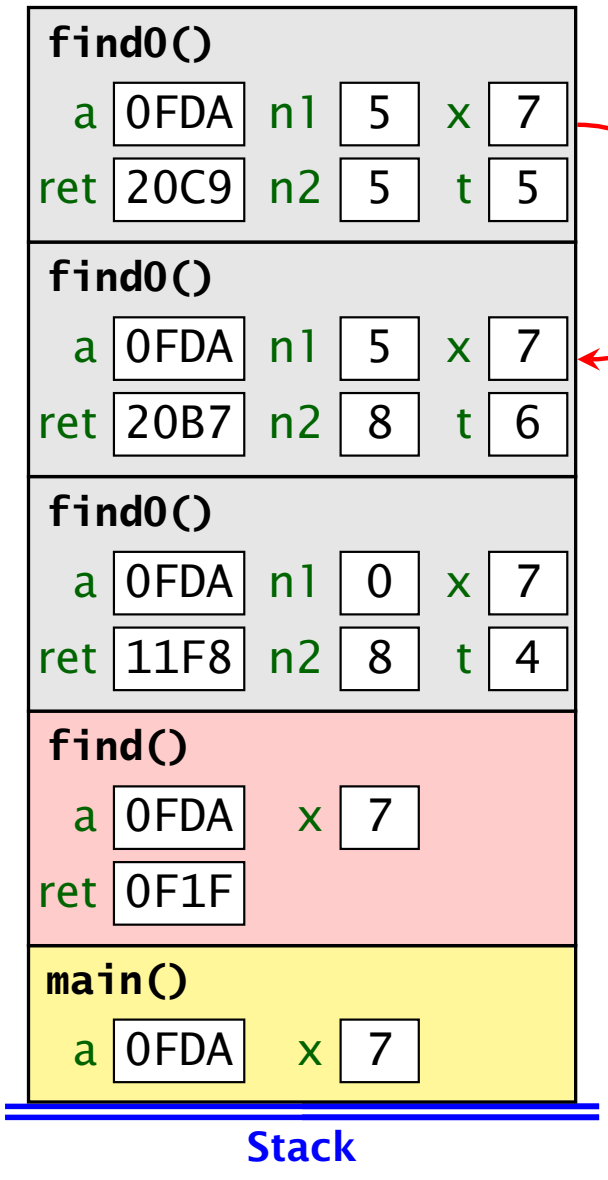

find0(a,x,n1,n2)
t=(n1+n2)/2
a[t]==x
no
yes
return t
n1==n2
no
yes
return -1
x>a[t]
no
yes
return find0(a,x,t+1,n2)
n1<t
no
yes
return -1
return find0(a,x,n1,t-1)

Ausführung

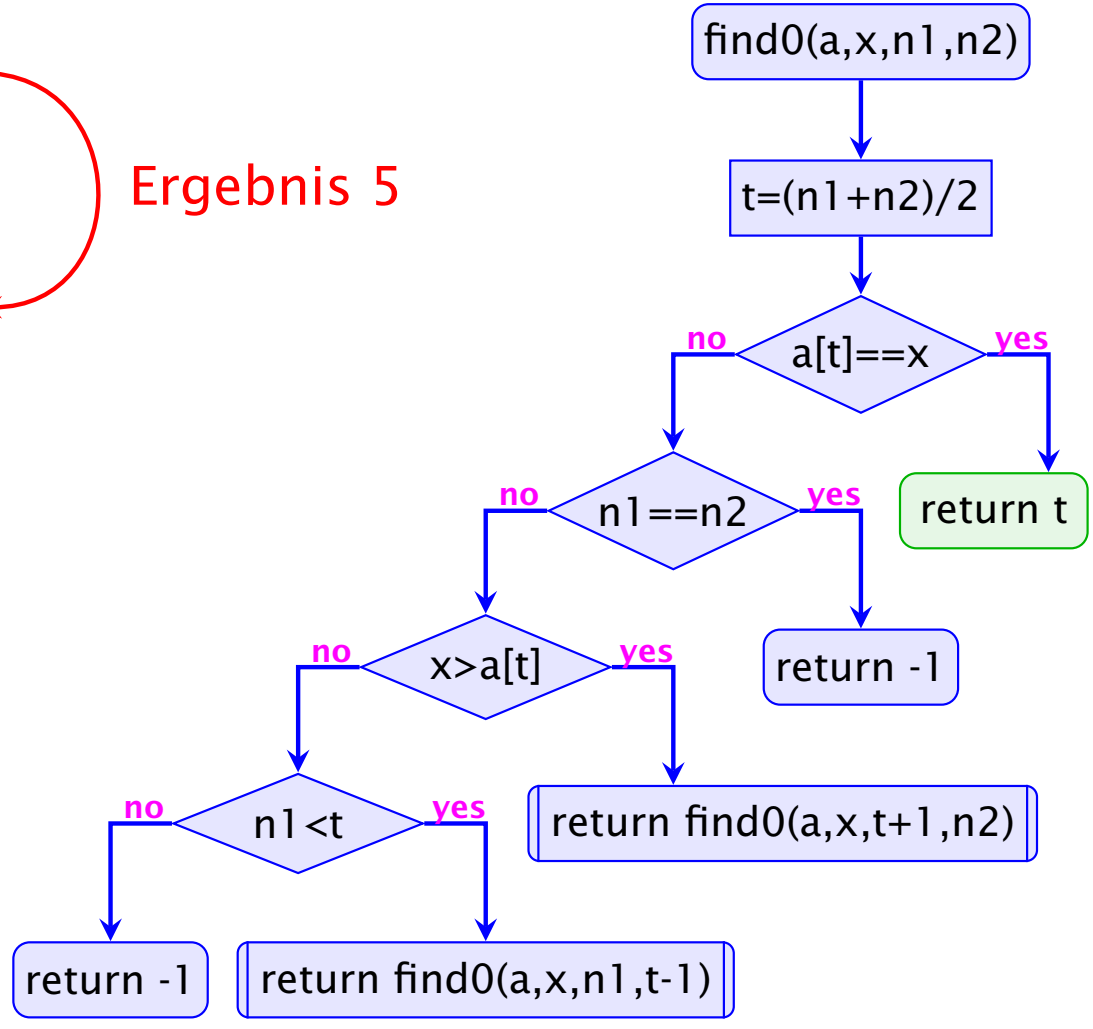

find0()
a 0FDA n1 5 x 7
ret 20C9 n2 5 t 5
find0()
a 0FDA n1 5 x 7
ret 20B7 n2 8 t 6
find0()
a 0FDA n1 0 x 7
ret 11F8 n2 8 t 4
find()
a 0FDA x 7
ret 0F1F
main()
a 0FDA x 7
Stack


find0(a,x,n1,n2)
t=(n1+n2)/2
a[t]==x
no
yes
return t
n1==n2
no
yes
return -1
x>a[t]
no
yes
return find0(a,x,t+1,n2)
n1<t
no
yes
return -1
return find0(a,x,n1,t-1)

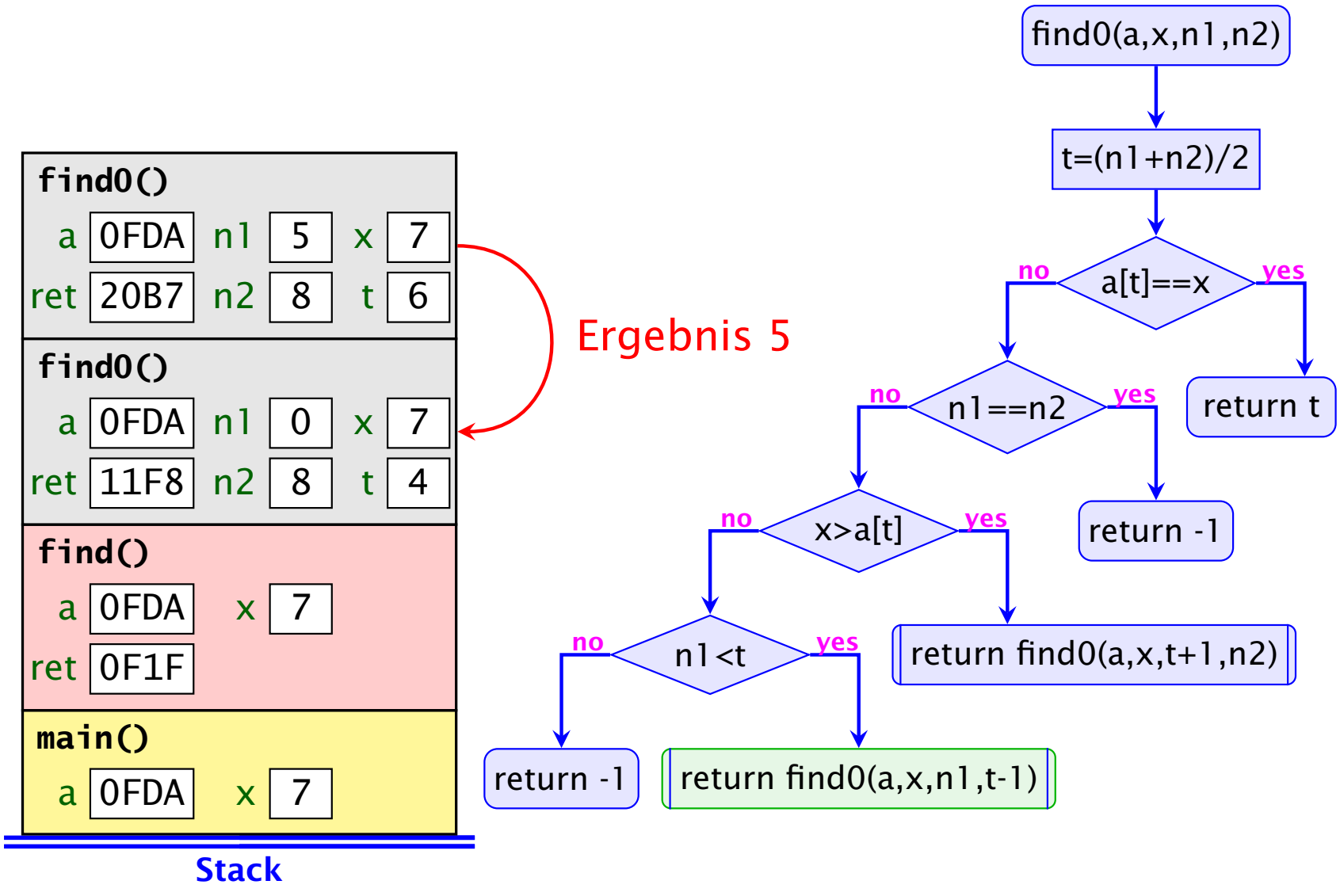
Ausführung



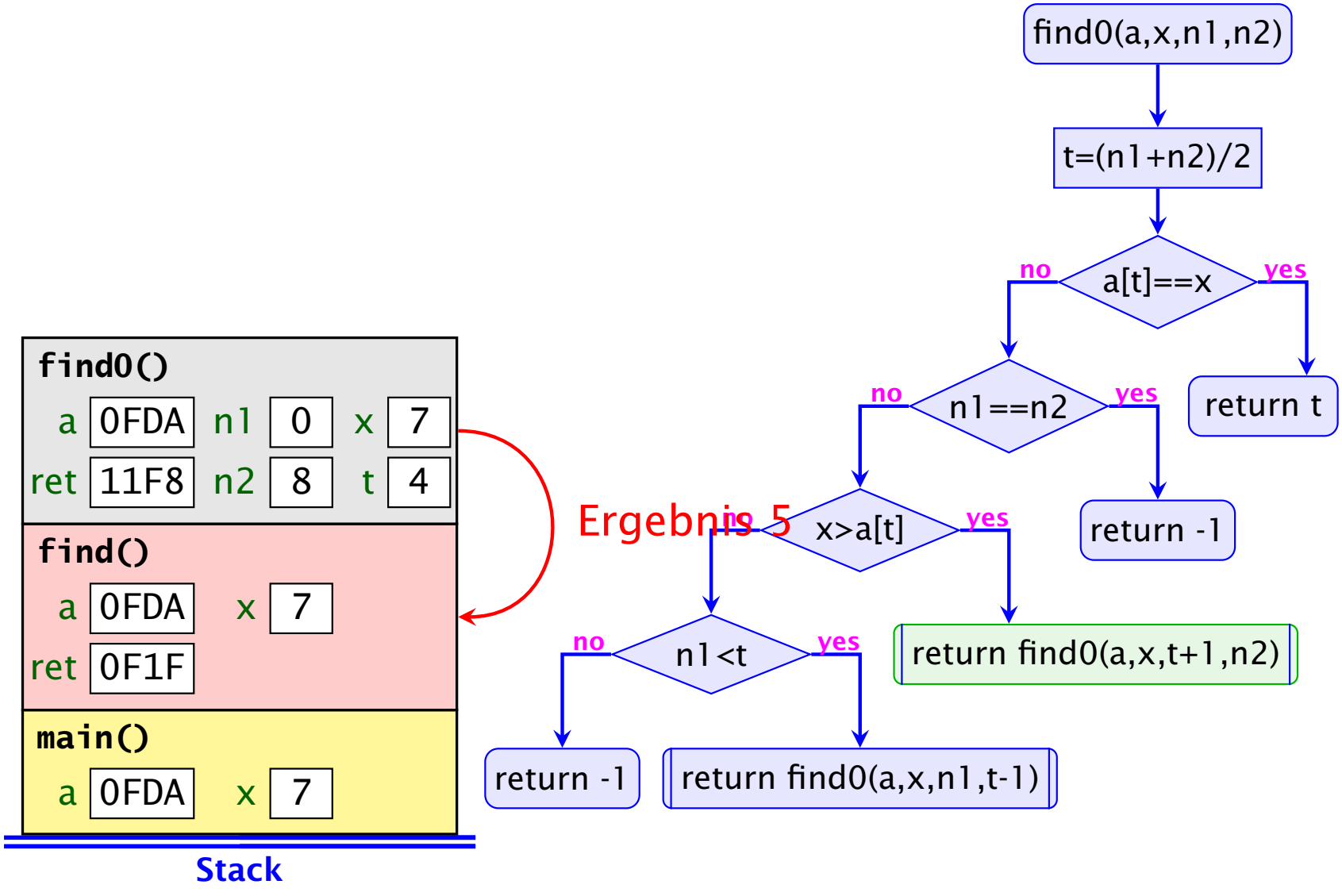
Ergebnis 5



Ausführung


find0()
a 0FDA n1 5 x 7
ret 20B7 n2 8 t 6
find0()
a 0FDA n1 0 x 7
ret 11F8 n2 8 t 4
find()
a 0FDA x 7
ret 0F1F
main()
a 0FDA x 7
Stack
Ergebnis 5
find0(a,x,n1,n2)
t=(n1+n2)/2
a[t]==x
no
yes
return t
n1==n2
no
yes
return -1
x>a[t]
no
yes
return find0(a,x,t+1,n2)
n1<t
no
yes
return -1
return find0(a,x,n1,t-1)

Ausführung


find0(a,x,n1,n2)
t=(n1+n2)/2
a[t]==x
no
yes
return t
n1==n2
no
yes
return -1
x>a[t]
no
yes
return find0(a,x,t+1,n2)
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return find0(a,x,n1,t-1)
Ergebnis 5
find0()
a 0FDA
n1 0
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ret 11F8
n2 8
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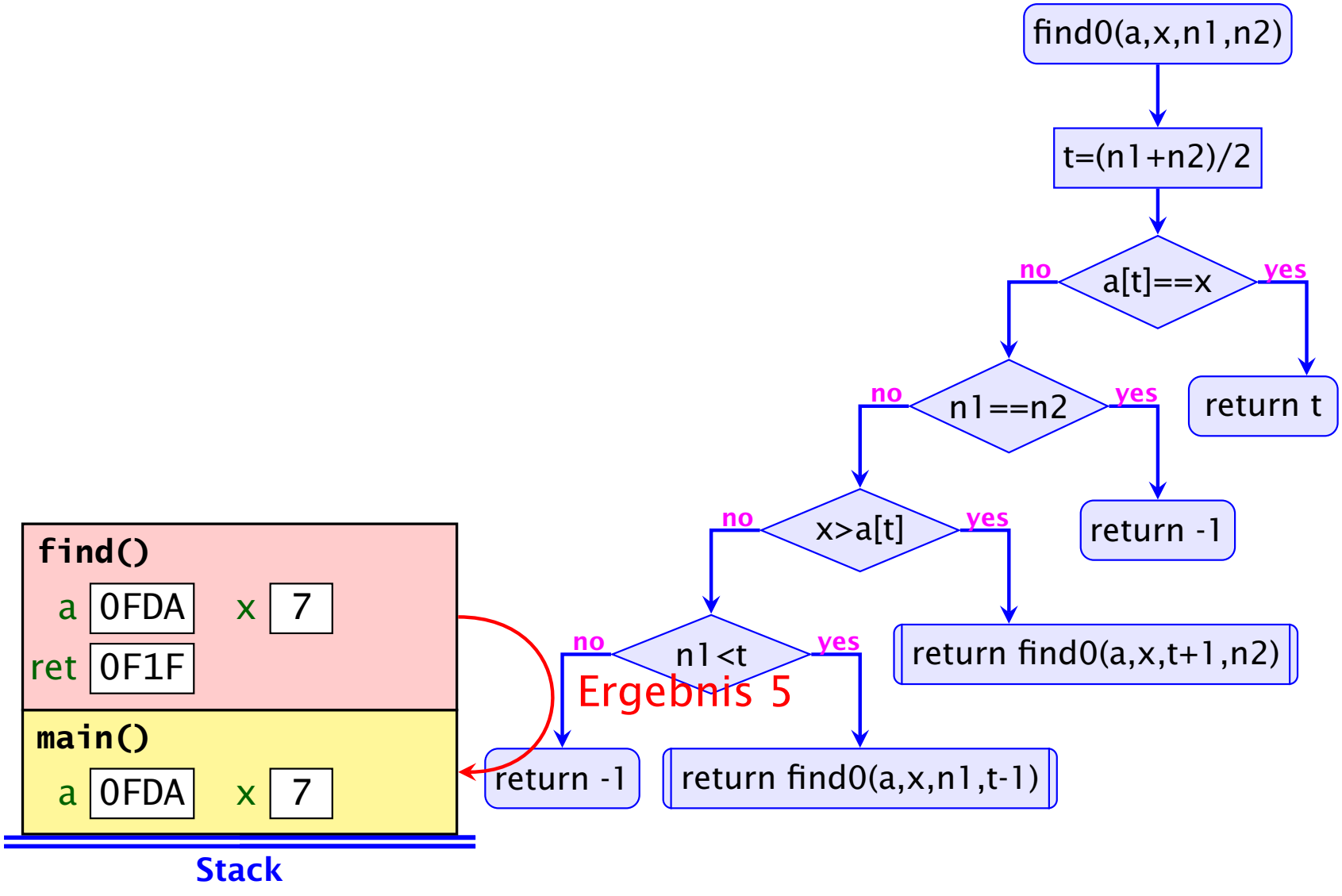
find()
a 0FDA
x 7
ret 0F1F
main()
a 0FDA
x 7
Stack

Ausführung


find0(a,x,n1,n2)
t=(n1+n2)/2
a[t]==x
no
yes
return t
n1==n2
no
yes
return -1
x>a[t]
no
yes
return find0(a,x,t+1,n2)
n1<t
no
yes
return -1
return find0(a,x,n1,t-1)
Ergebnis 5
find()
a 0FDA x 7
ret 0F1F
main()
a 0FDA x 7
Stack

Terminierung

Um zu beweisen, dass `find0()` terminiert, beobachten wir:

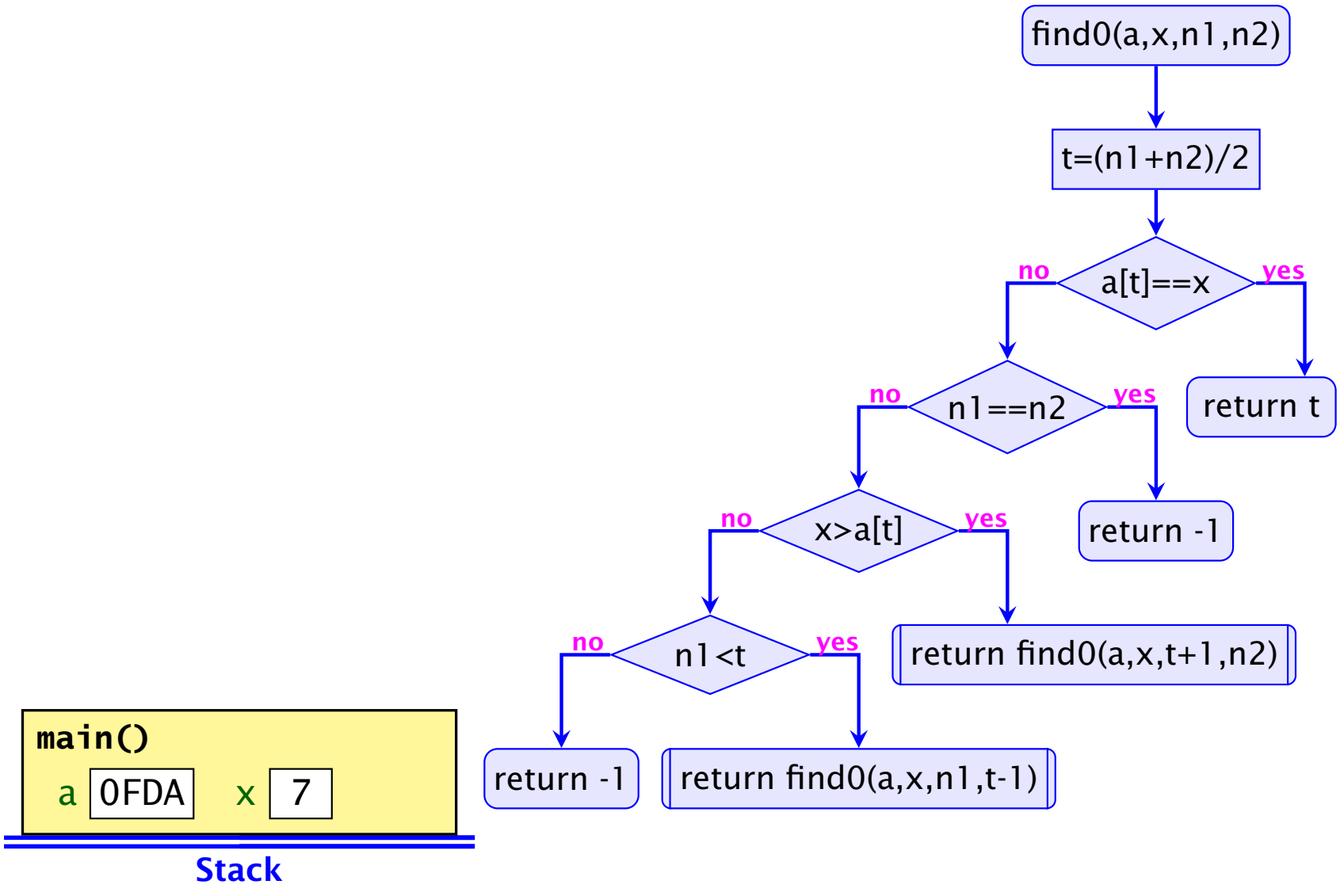
1. Wird `find0()` für ein einelementiges Intervall `[n,n]` aufgerufen, dann terminiert der Funktionsaufruf direkt.
2. wird `find0()` für ein Intervall `[n1,n2]` aufgerufen mit mehr als einem Element, dann terminiert der Aufruf entweder direkt (weil `x` gefunden wurde), oder `find0()` wird mit einem Intervall aufgerufen, das echt in `[n1,n2]` enthalten ist, genauer: sogar maximal die Hälfte der Elemente von `[n1,n2]` enthält.

Ähnliche Beweistechnik wird auch für andere rekursive Funktionen verwendet.

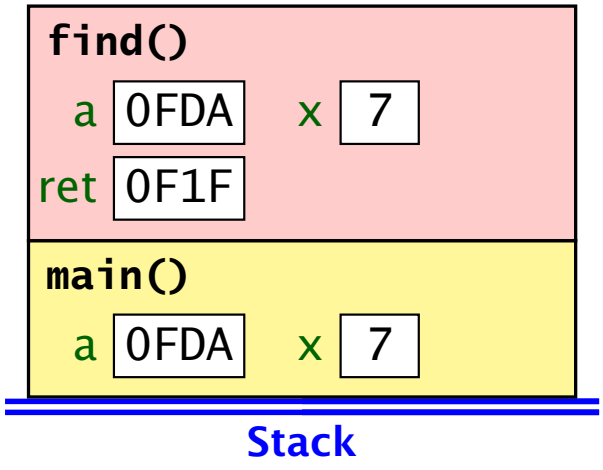
Beobachtung

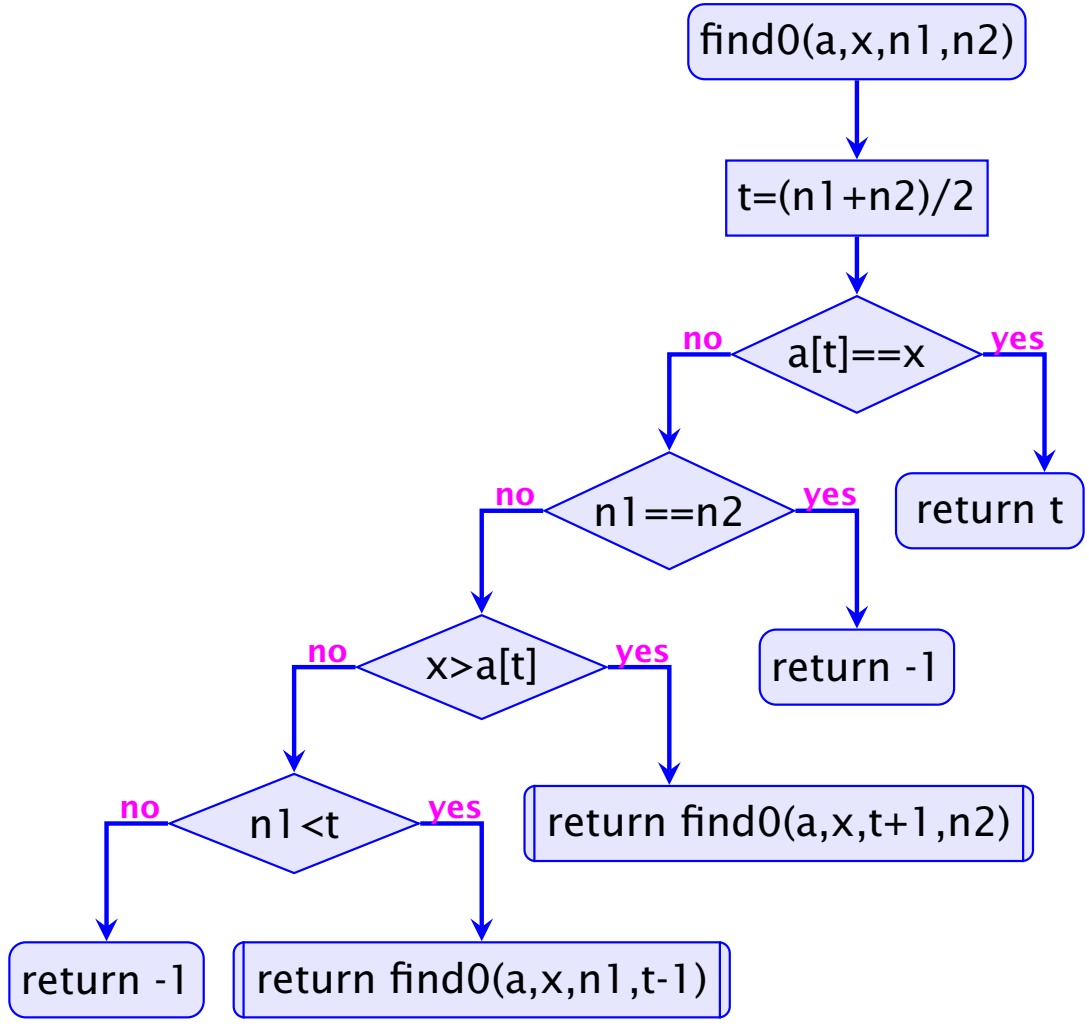
- Das Ergebnis eines Aufrufs von `find0()` liefert direkt das Ergebnis auch für die aufrufende Funktion!
- Solche Rekursion heißt End- oder Tail-Rekursion.
- End-Rekursion kann auch ohne Aufrufkeller implementiert werden...
- **Idee:** lege den neuen Aufruf von `find0()` nicht oben auf den Stapel drauf, sondern ersetze den bereits dort liegenden Aufruf!

Verbesserte Ausführung

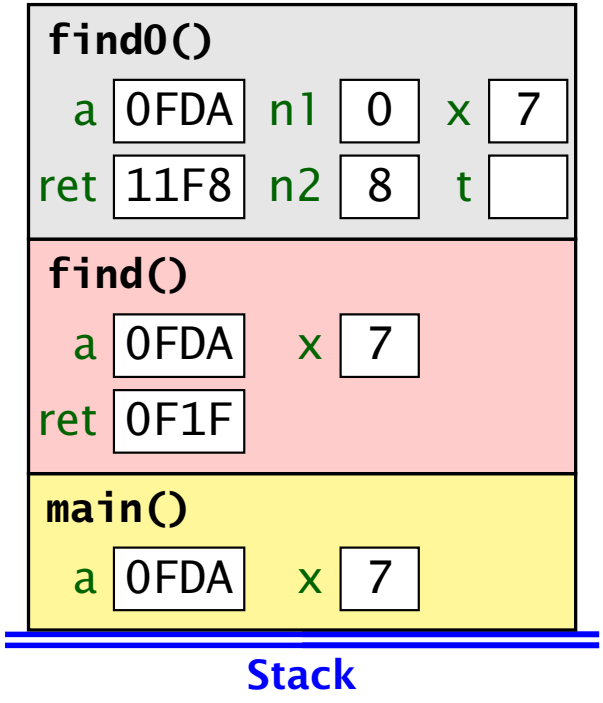


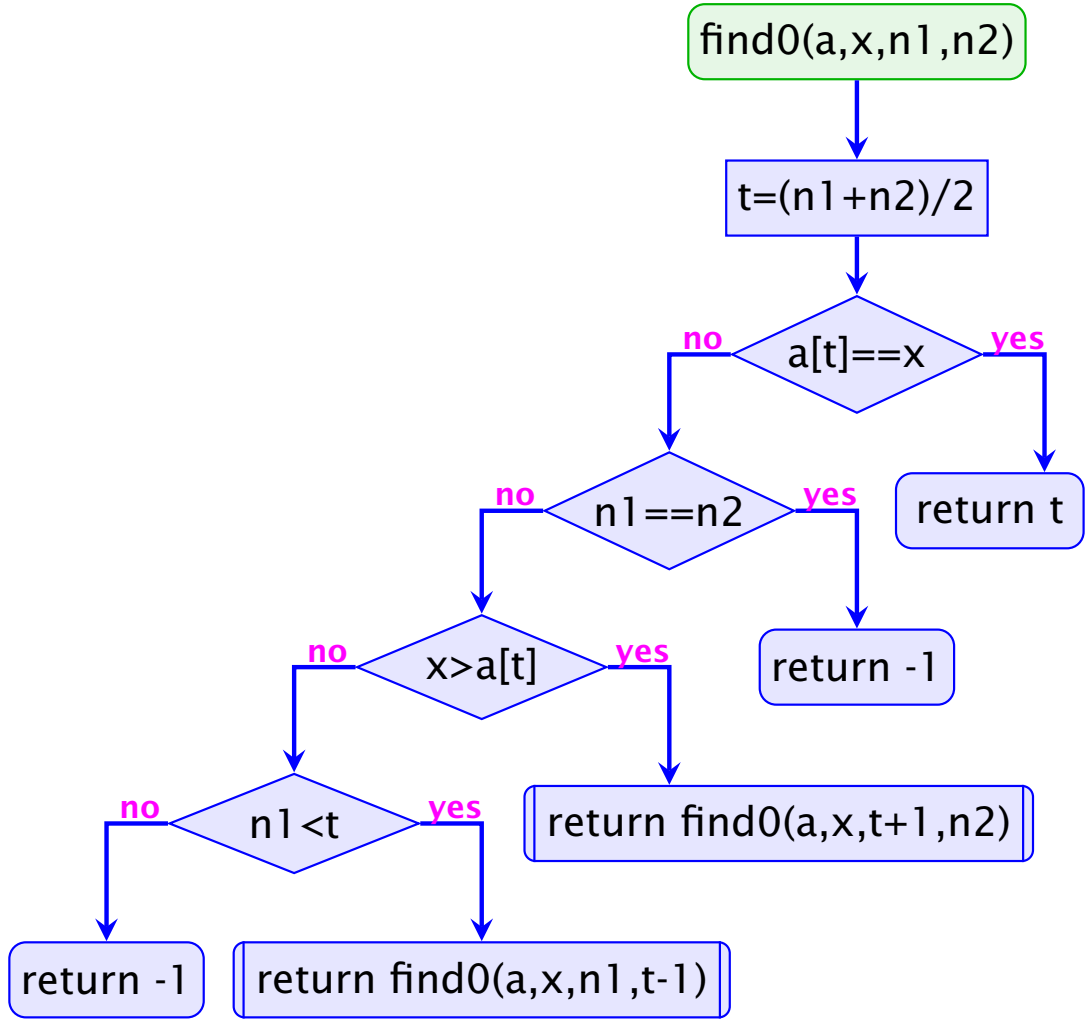
Verbesserte Ausführung



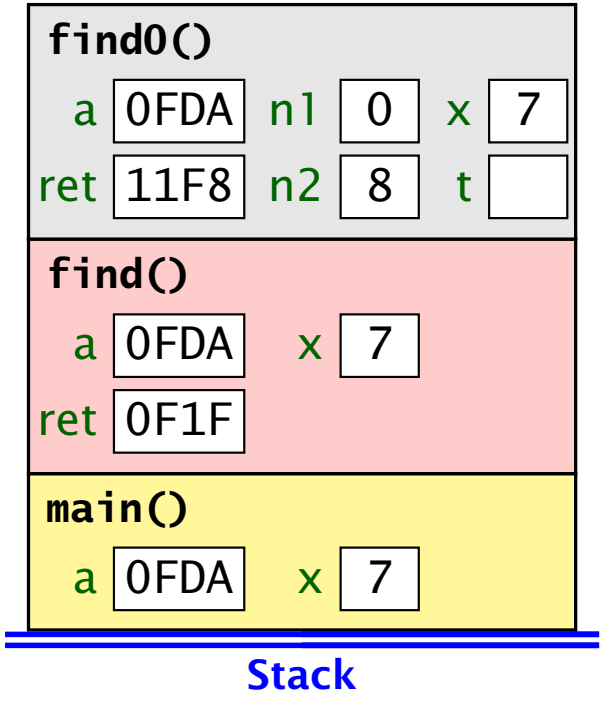


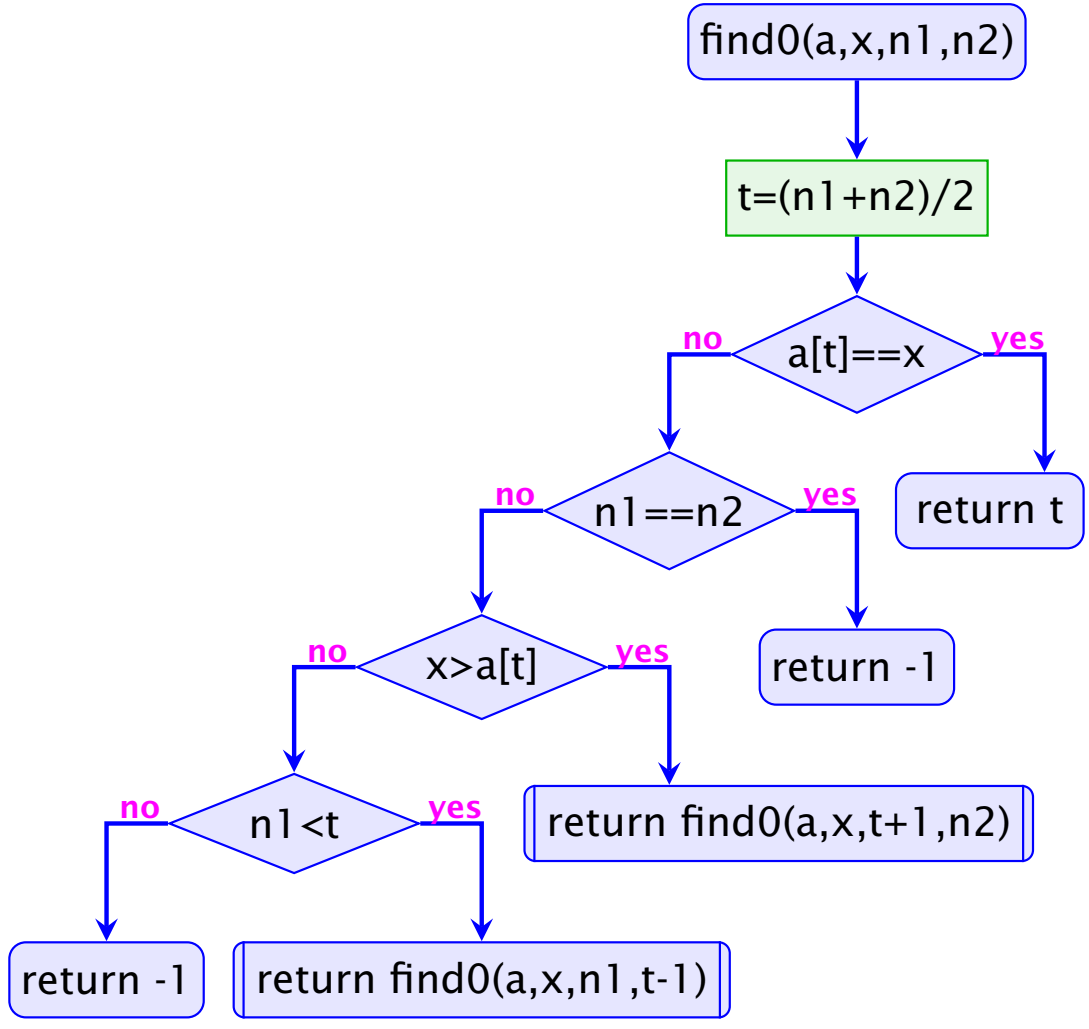
Verbesserte Ausführung



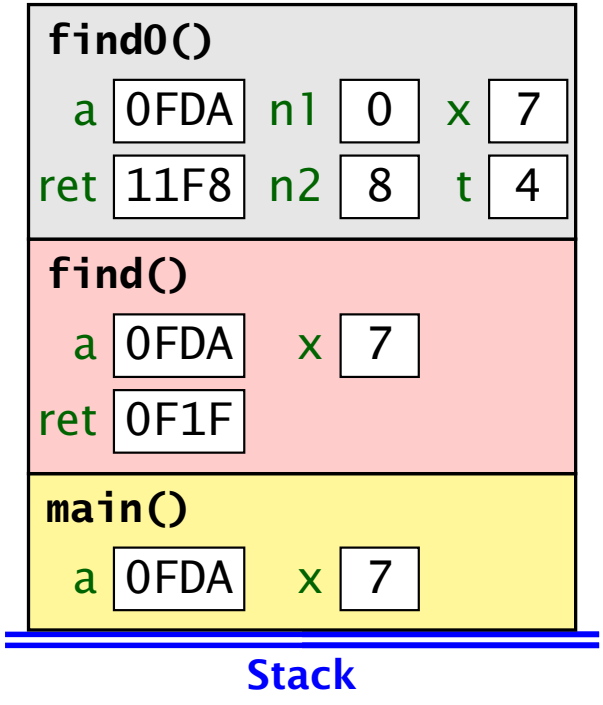


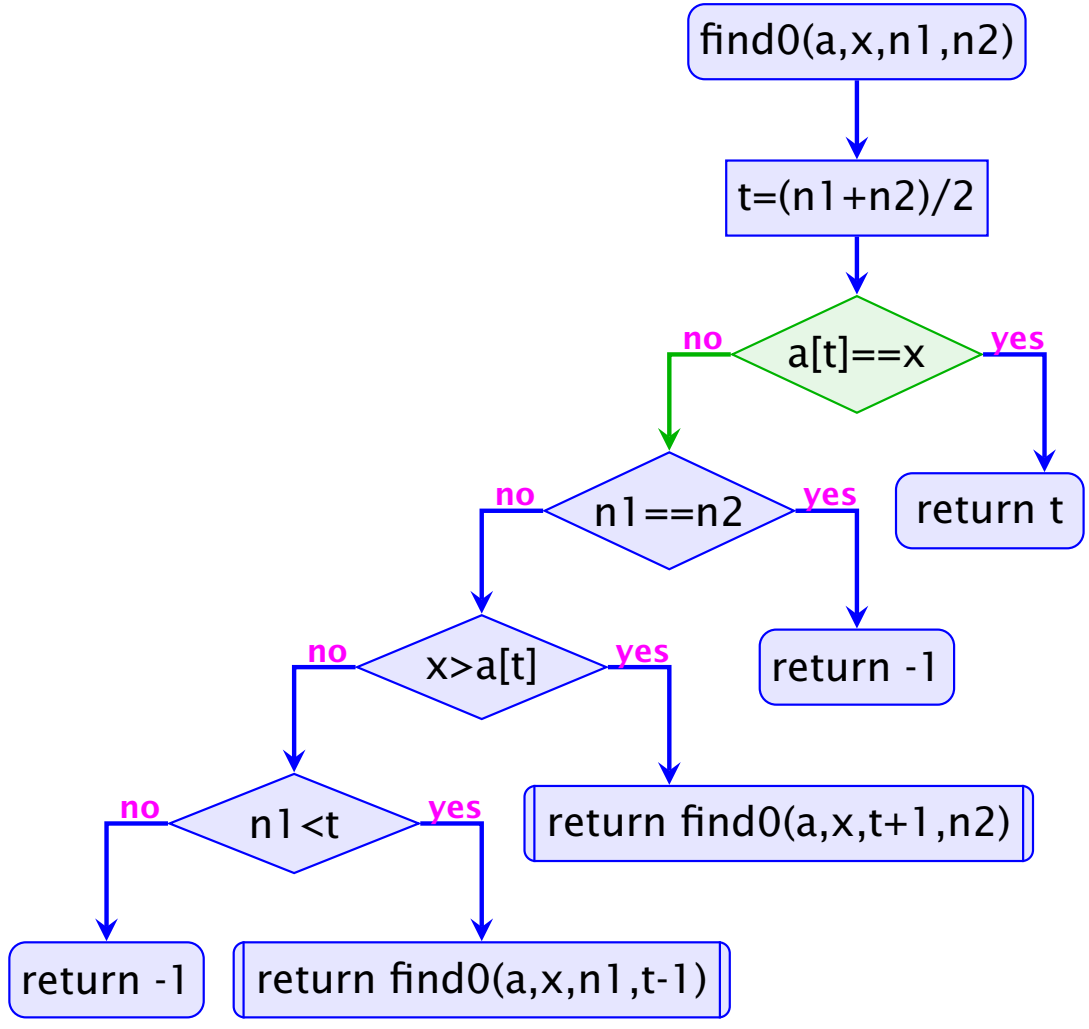
Verbesserte Ausführung



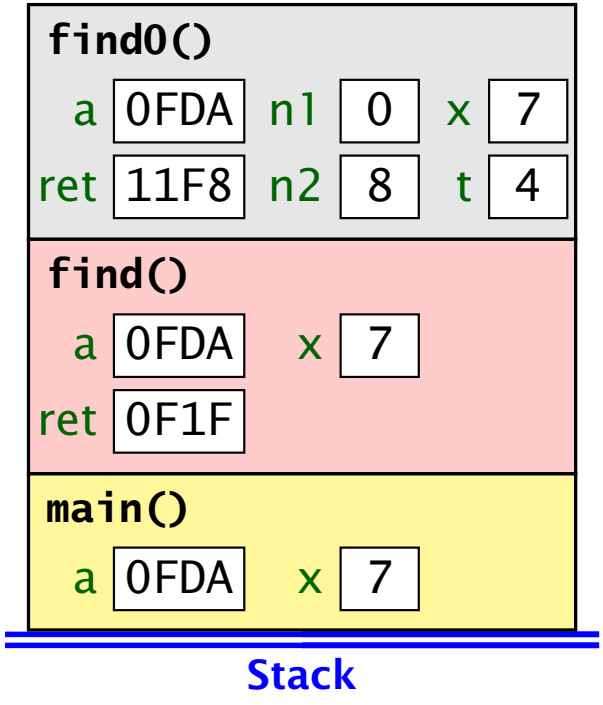


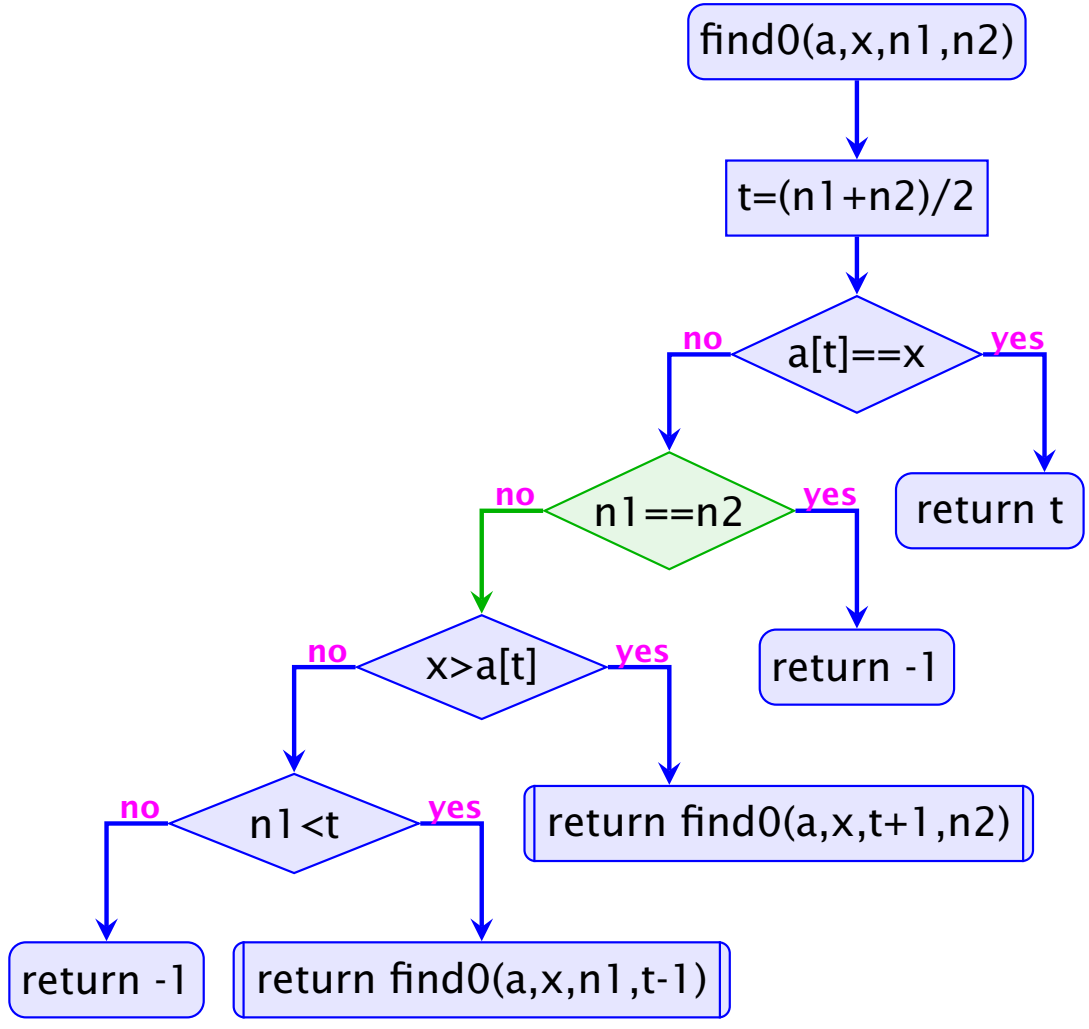
Verbesserte Ausführung



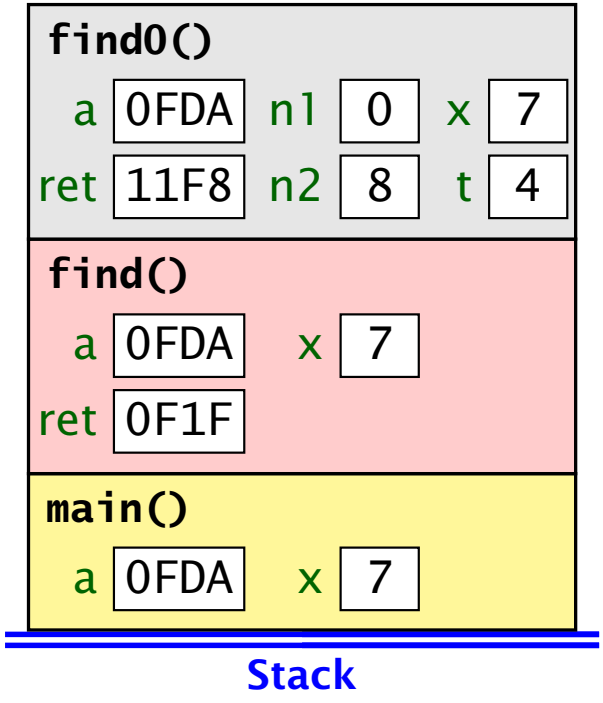


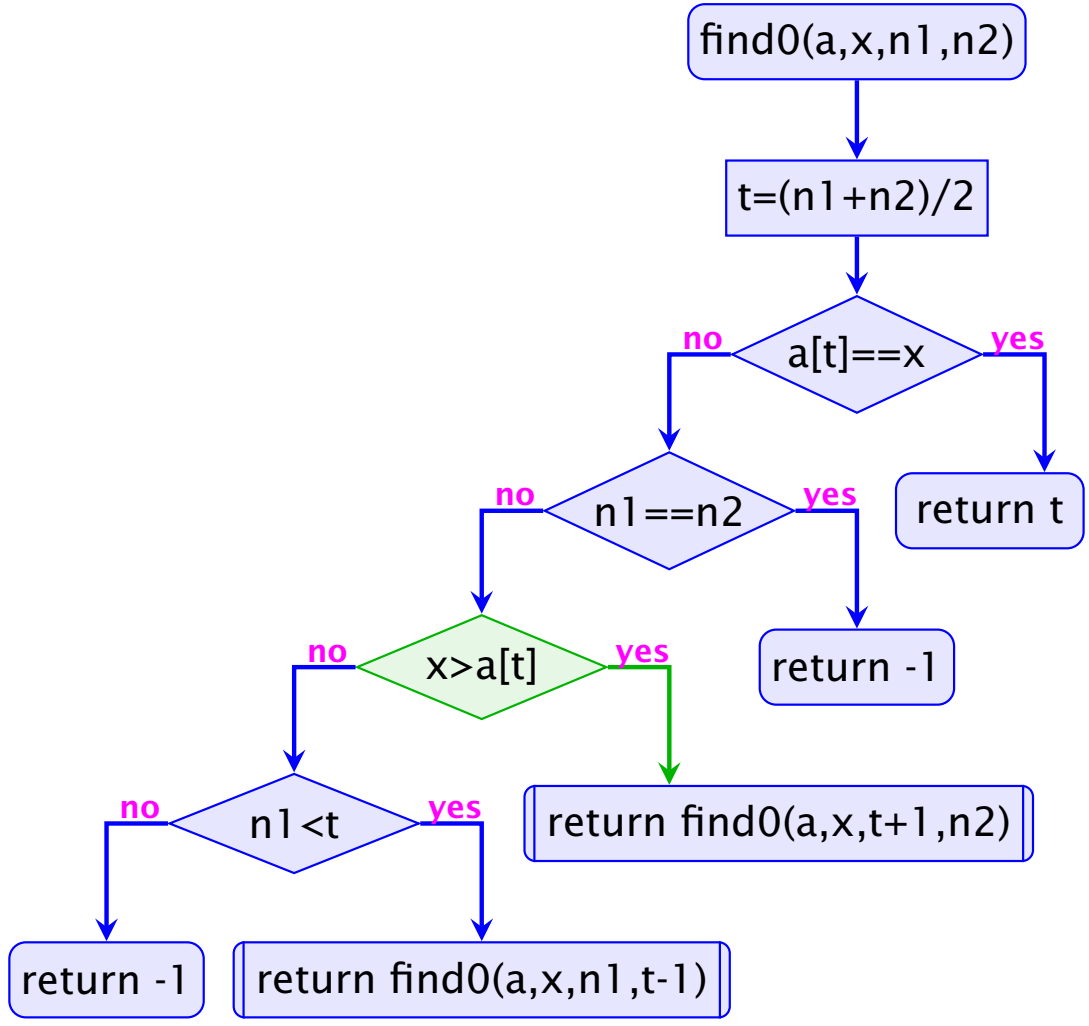
Verbesserte Ausführung





Verbesserte Ausführung





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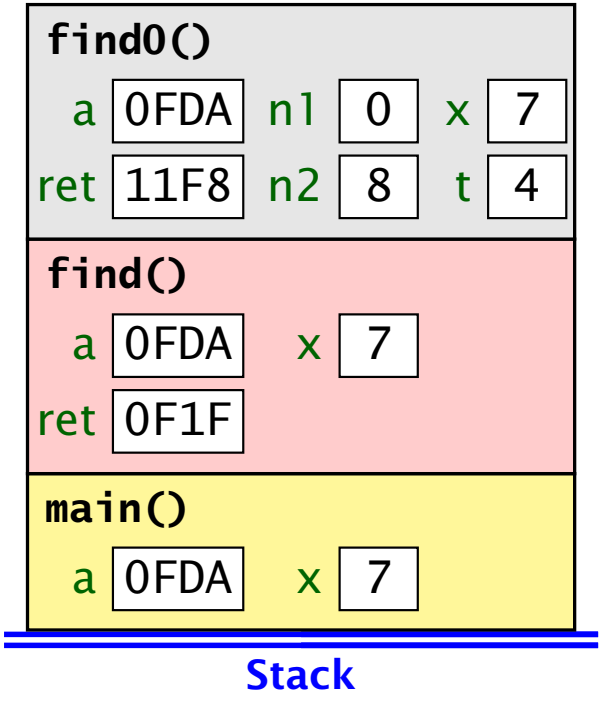

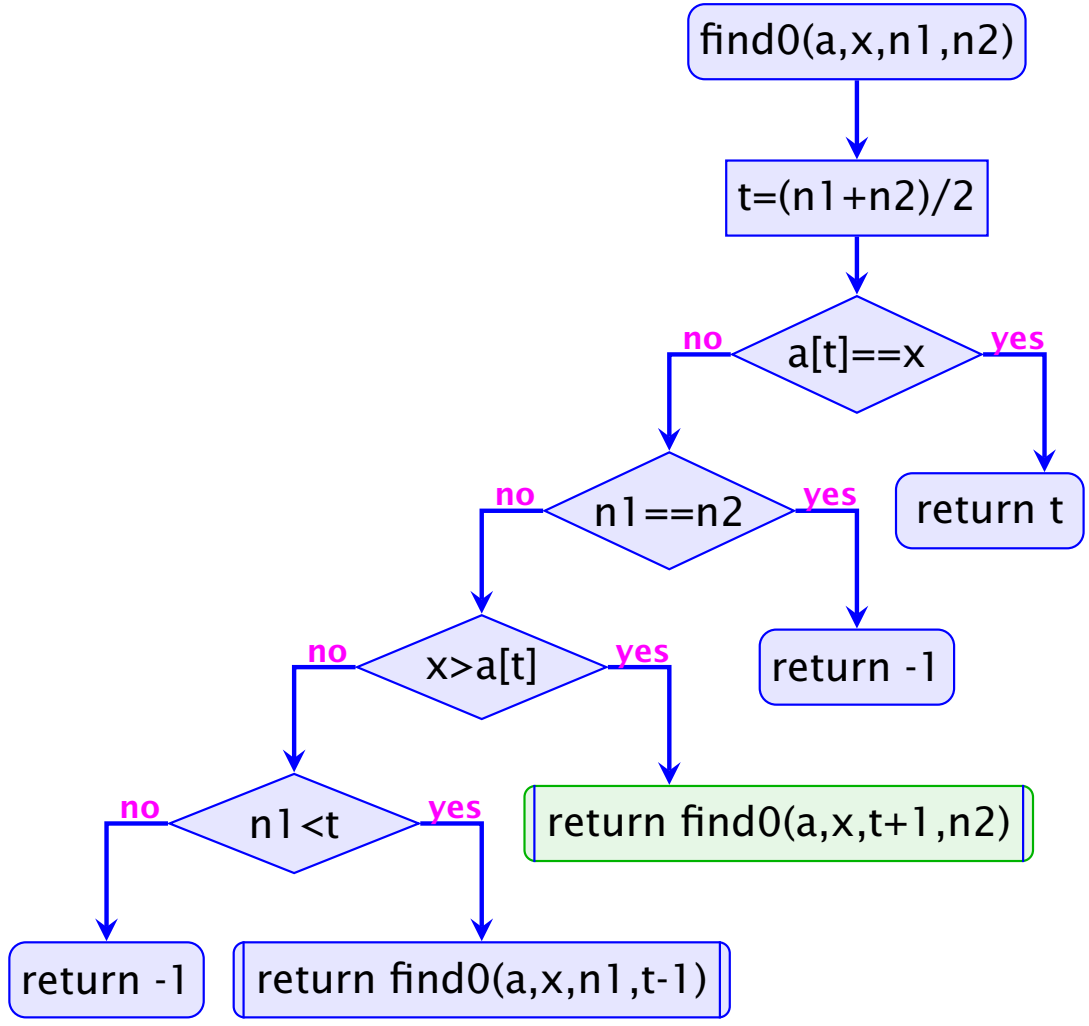

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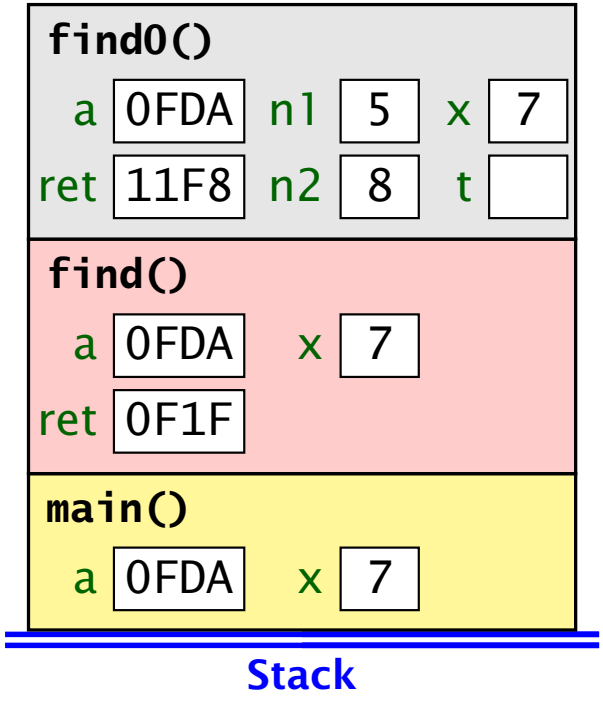

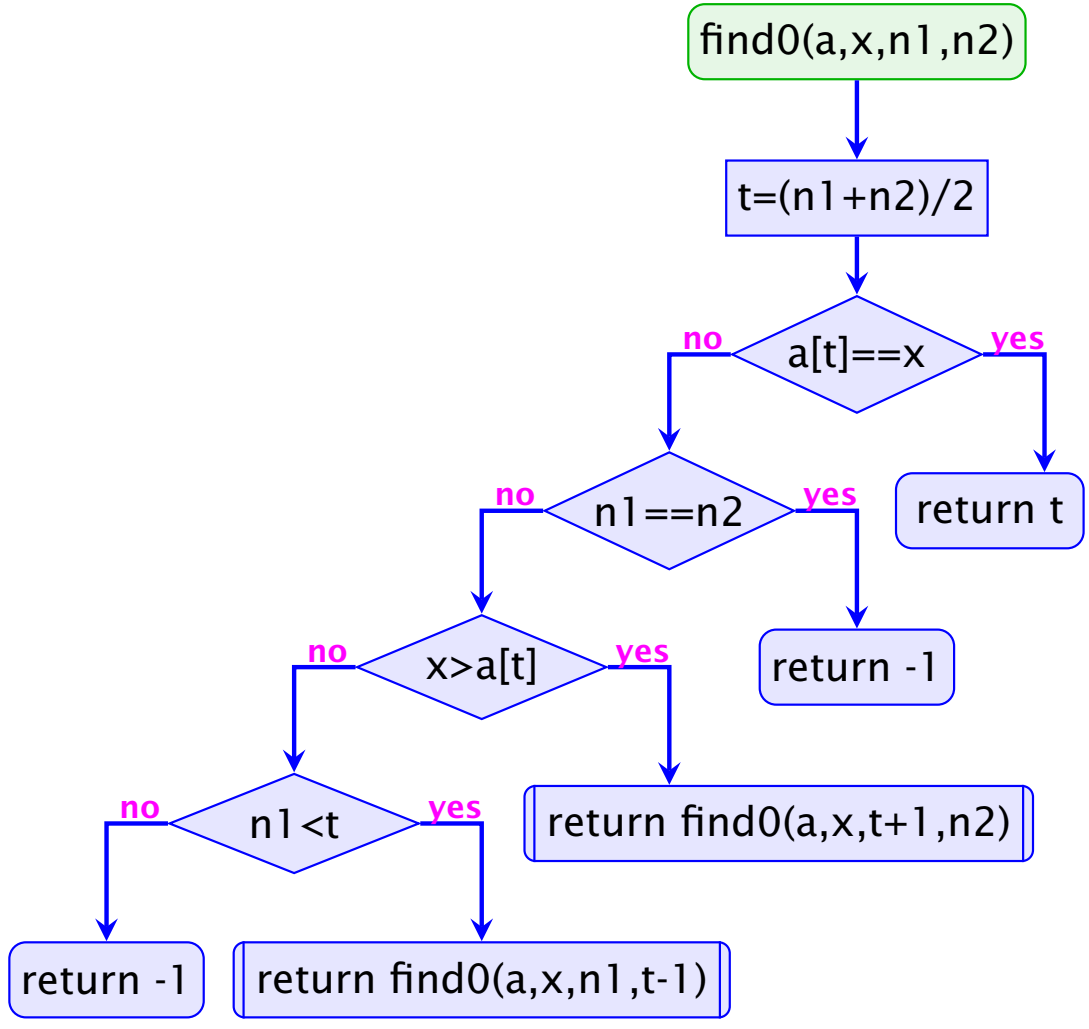

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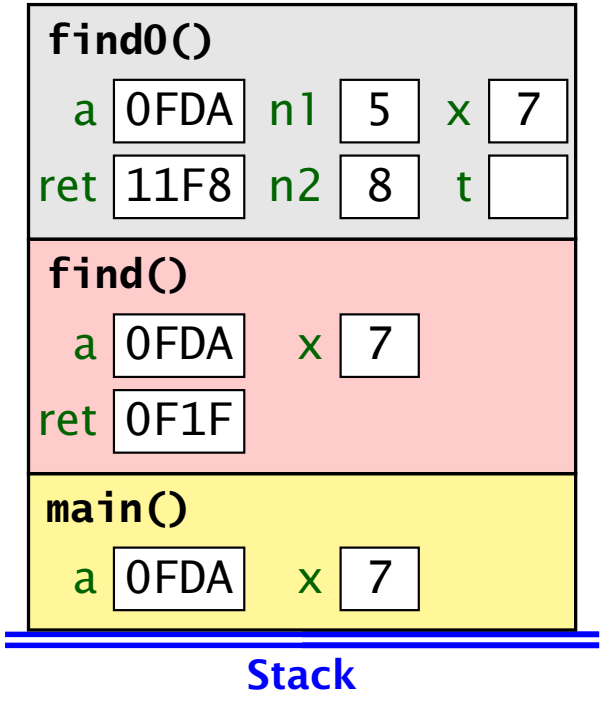

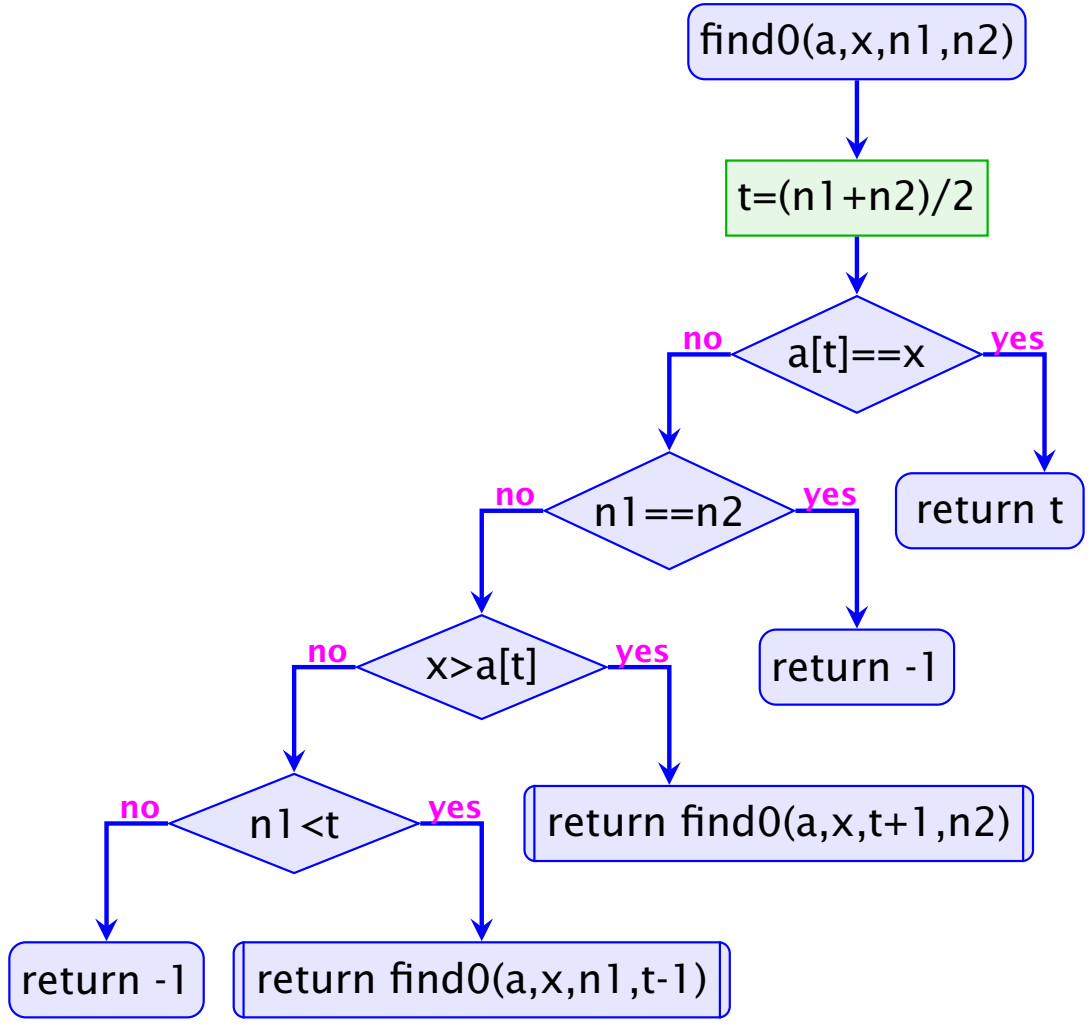

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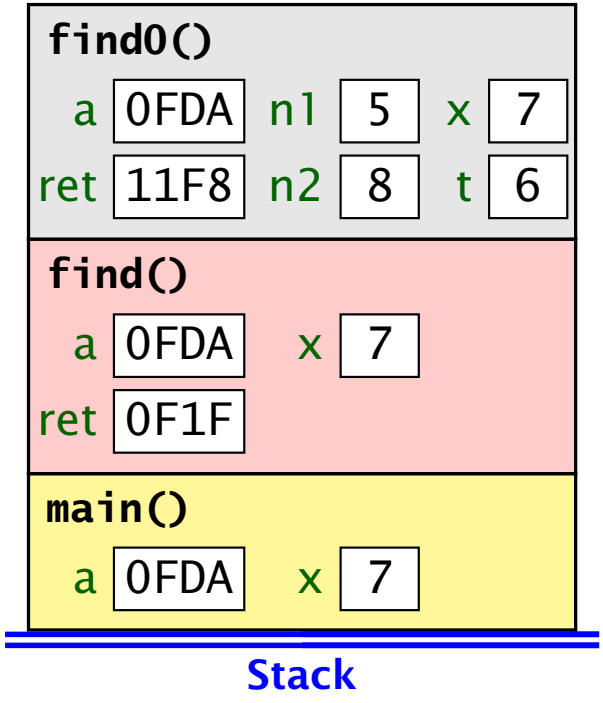

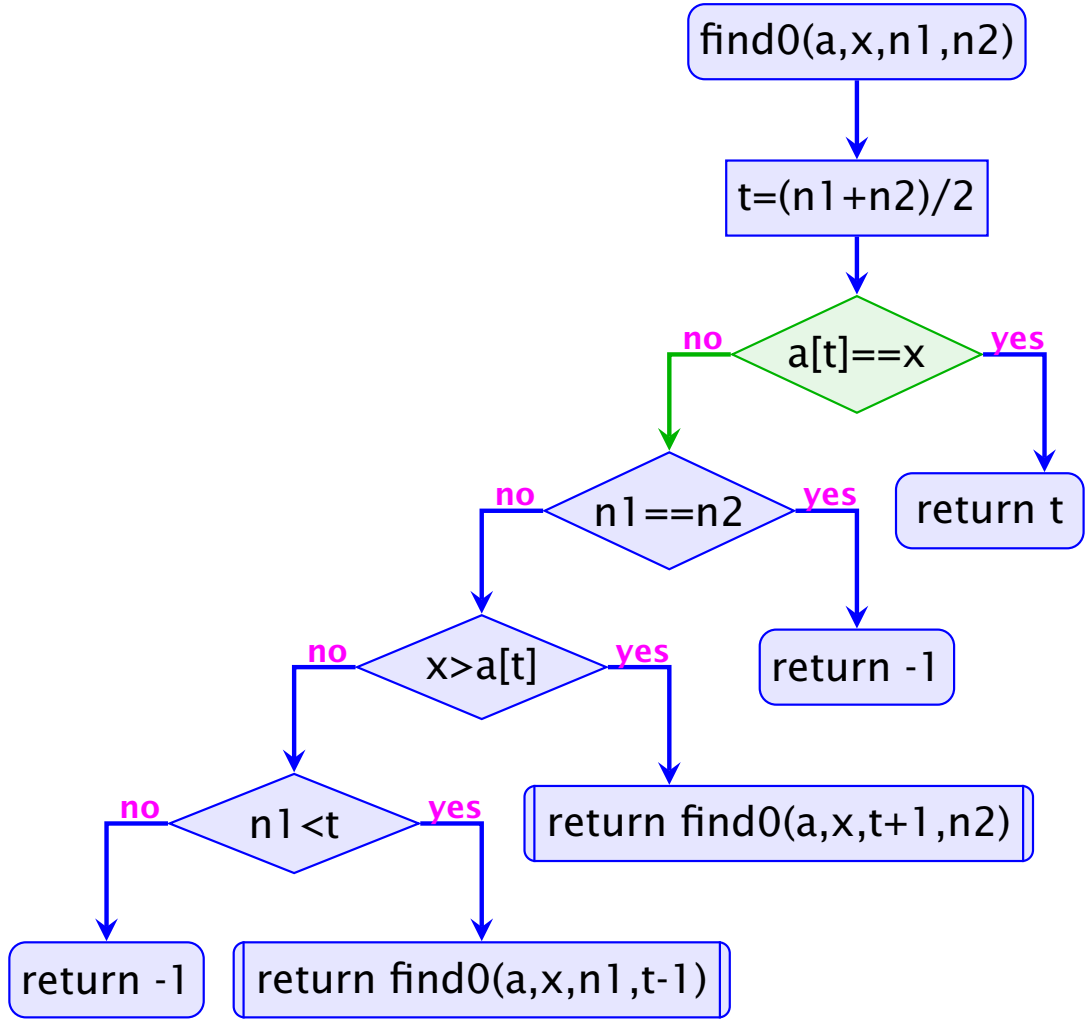

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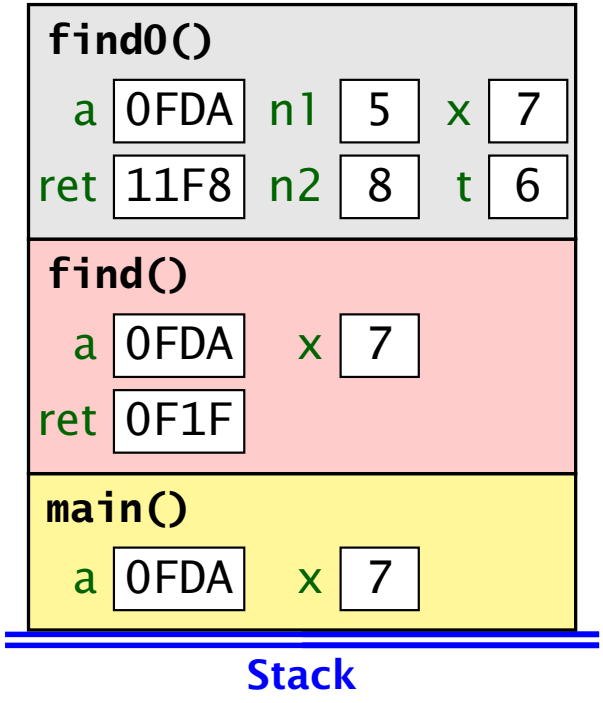

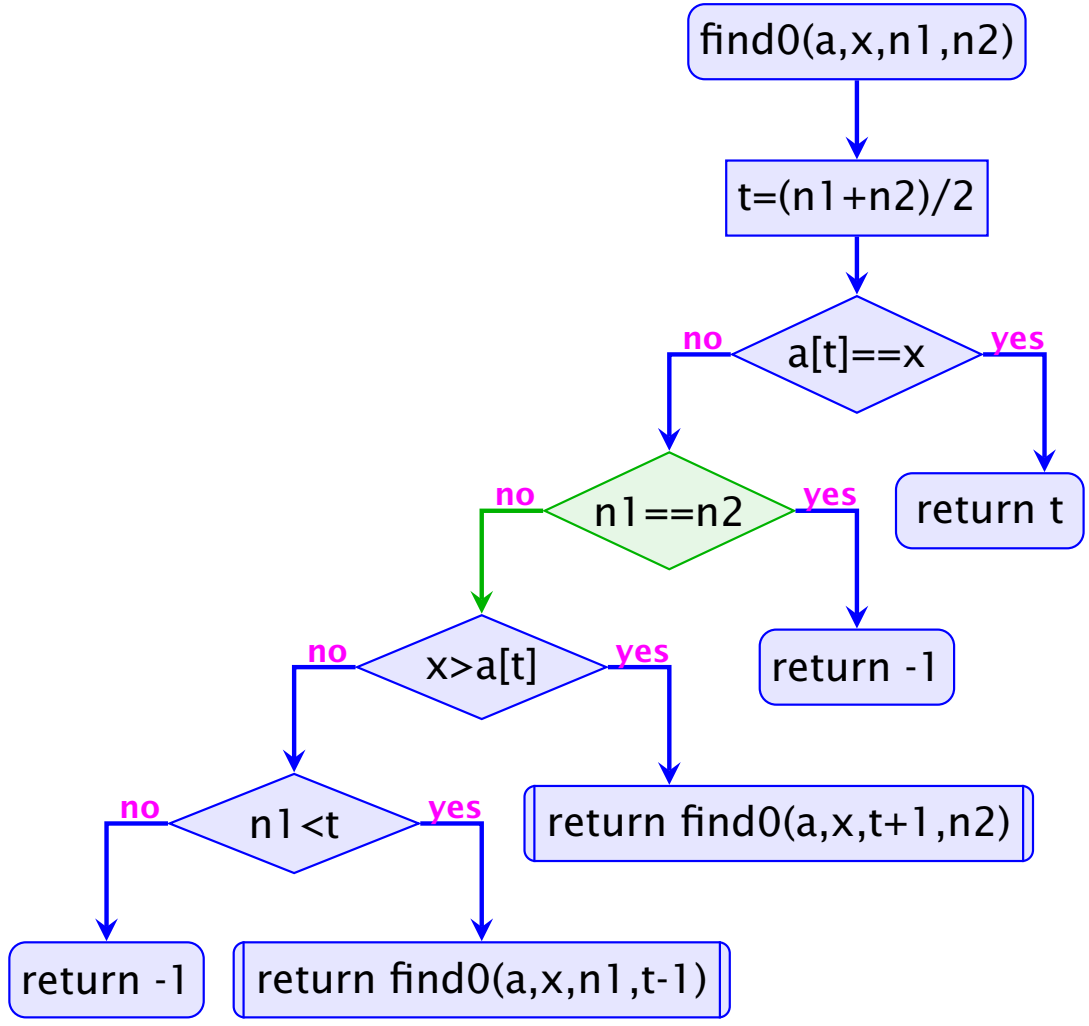

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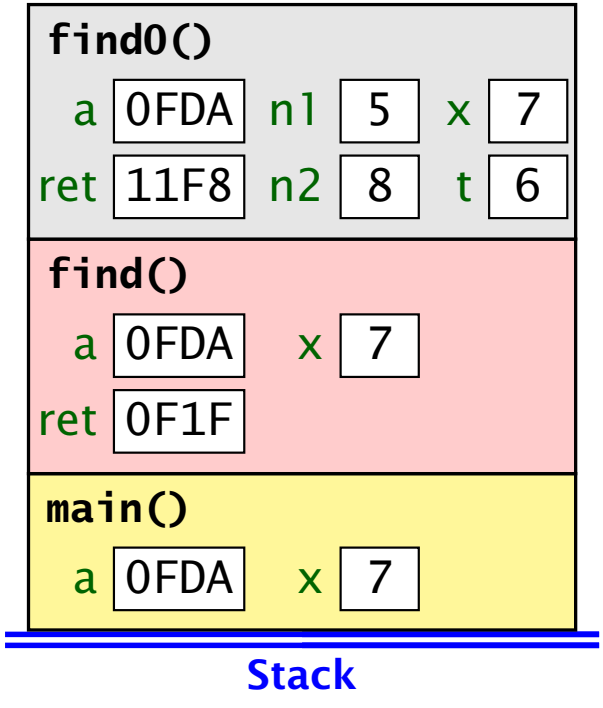

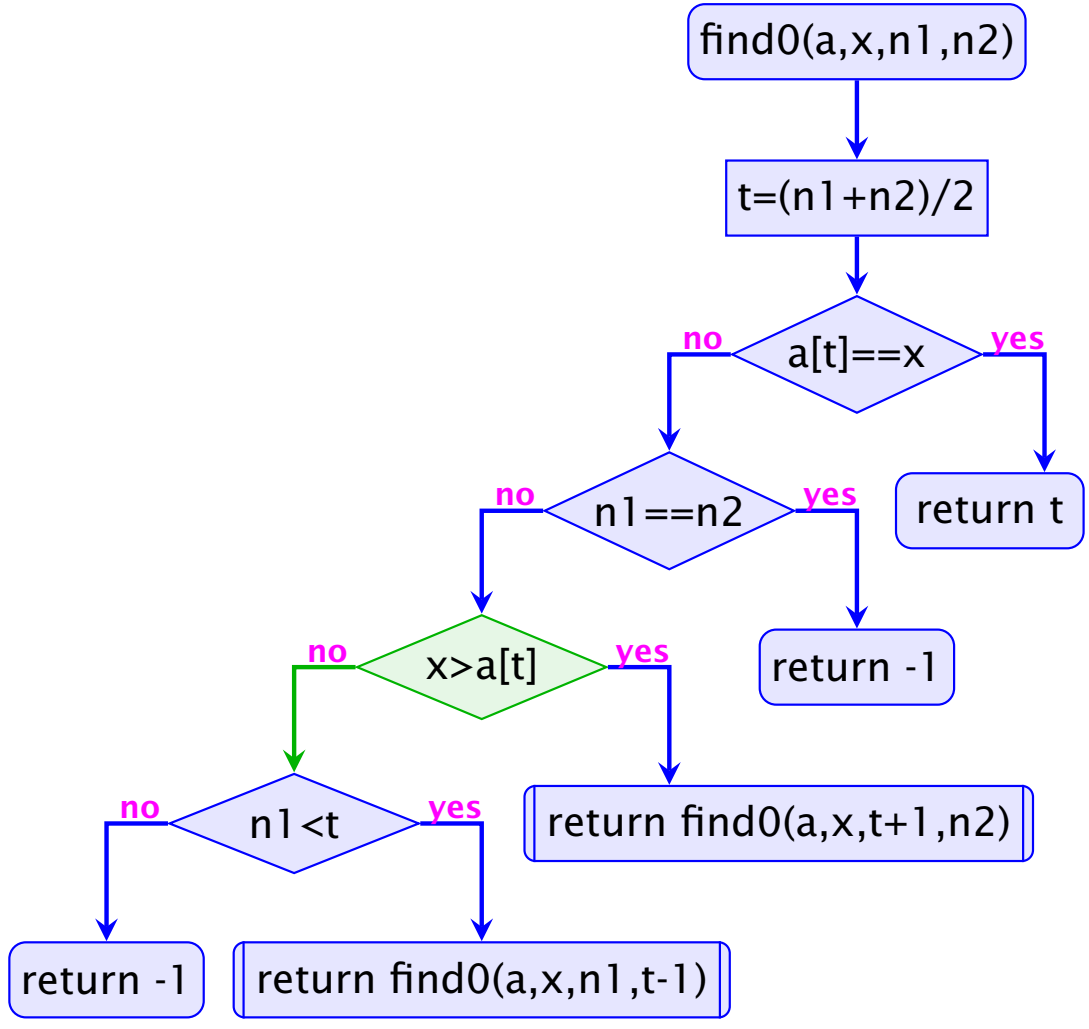

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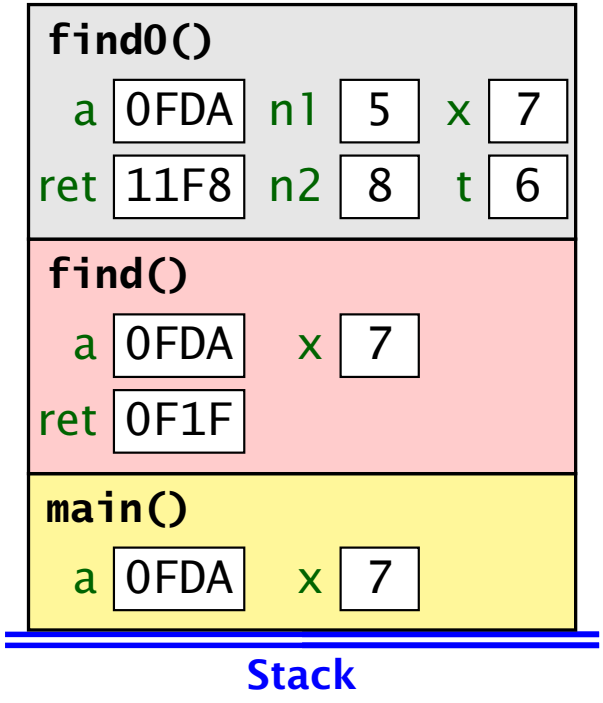

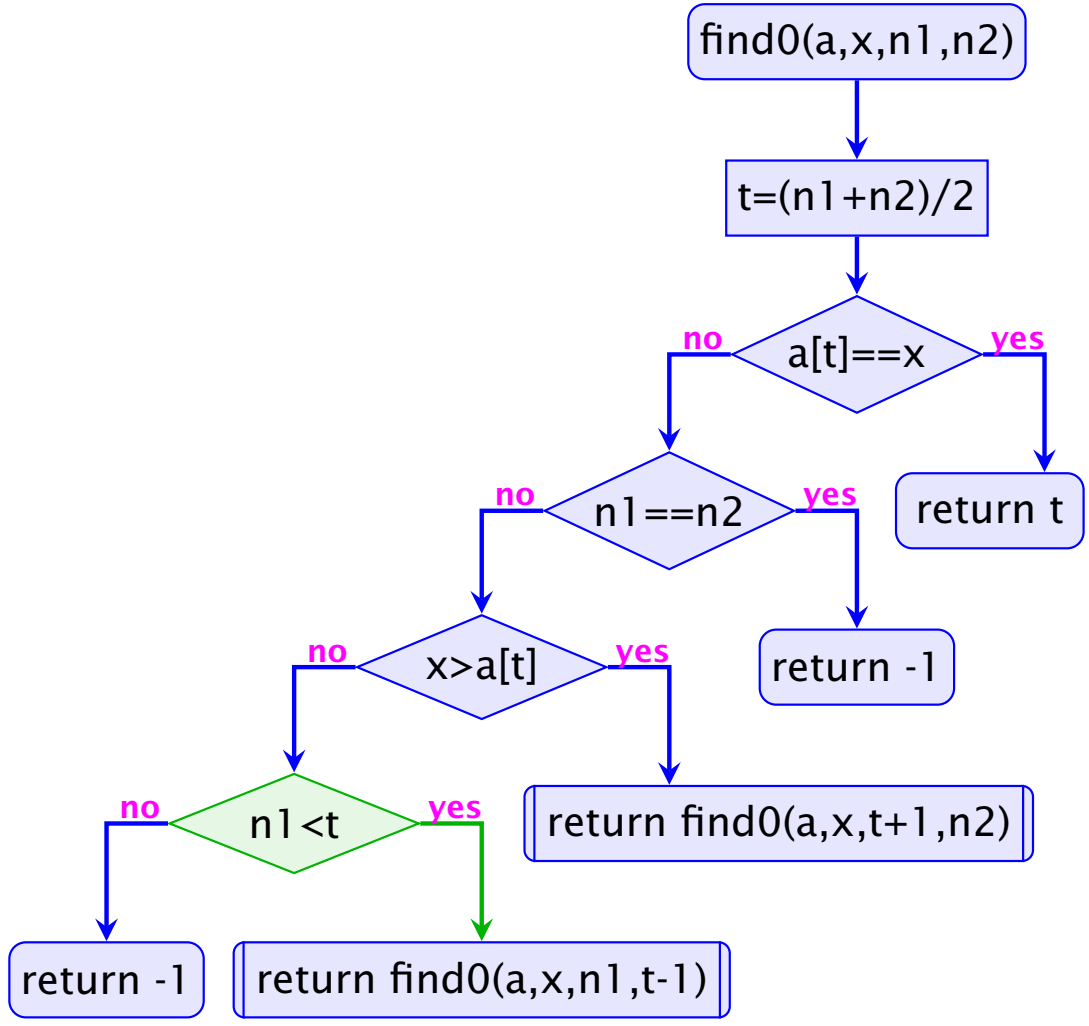

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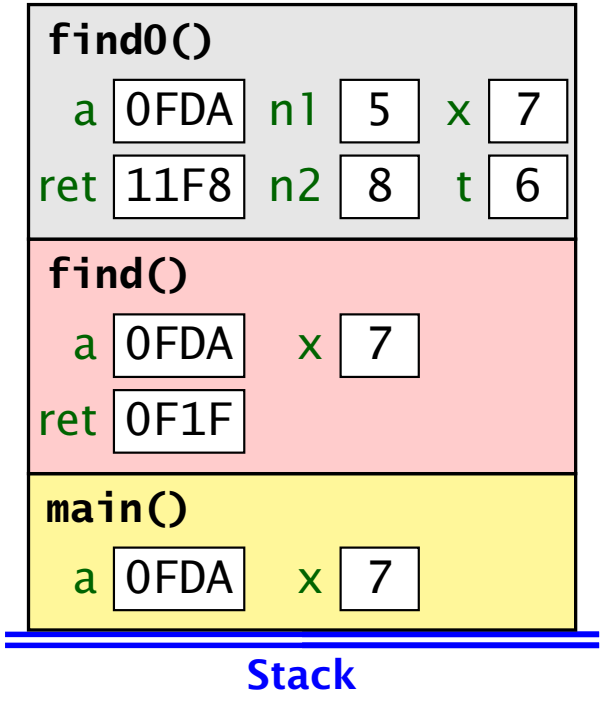

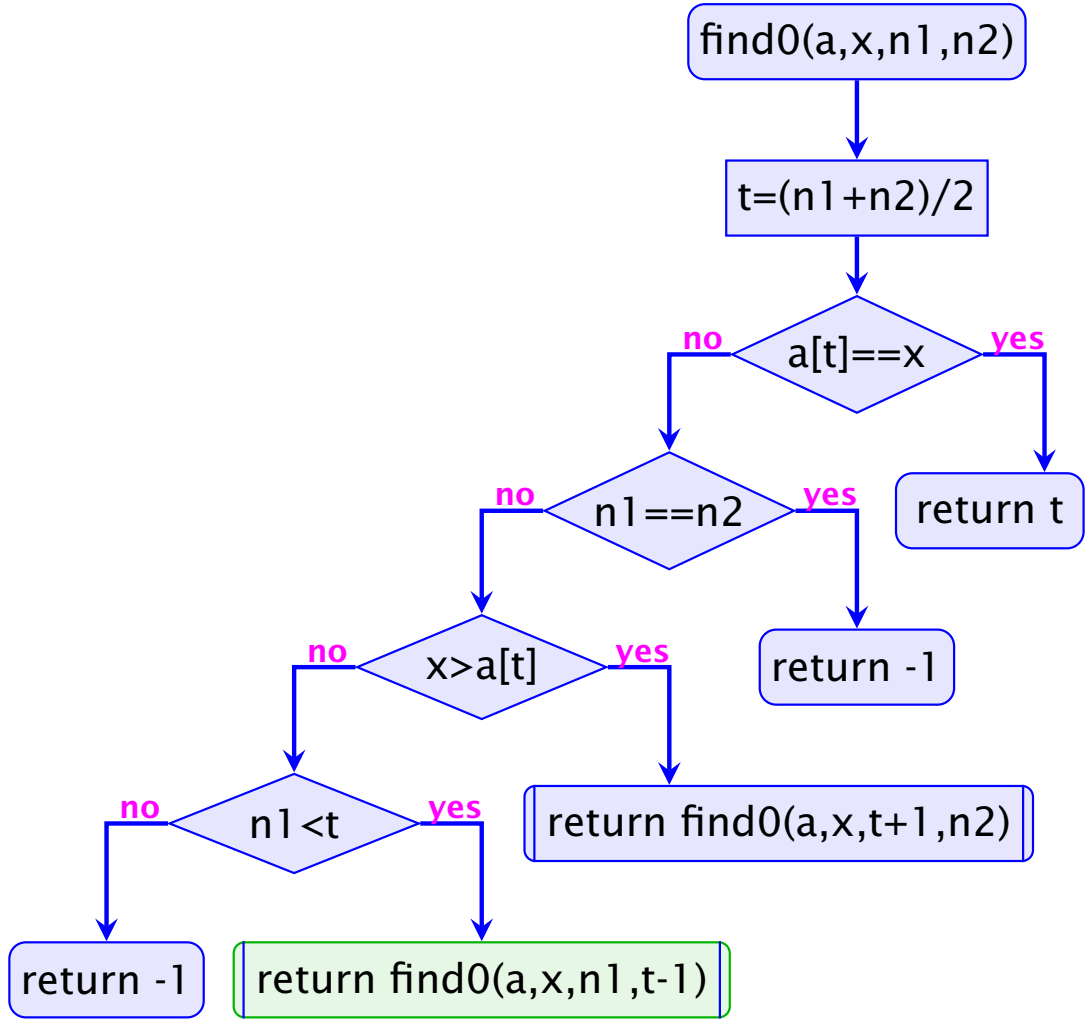

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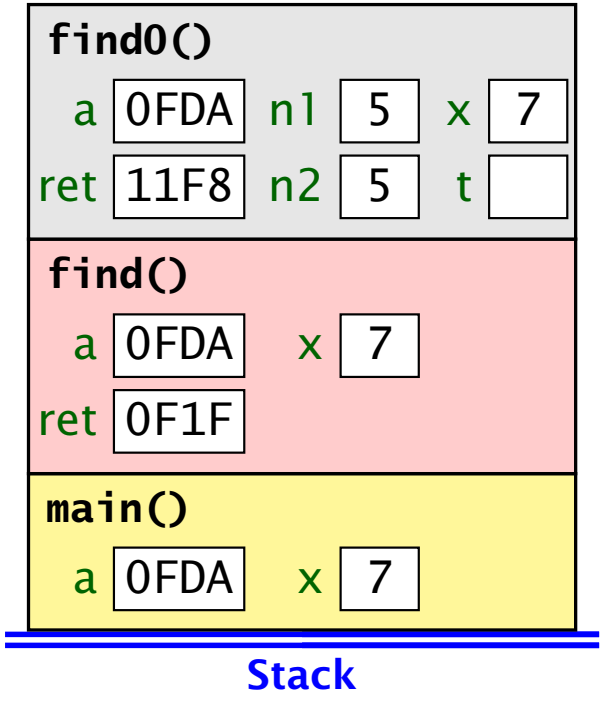

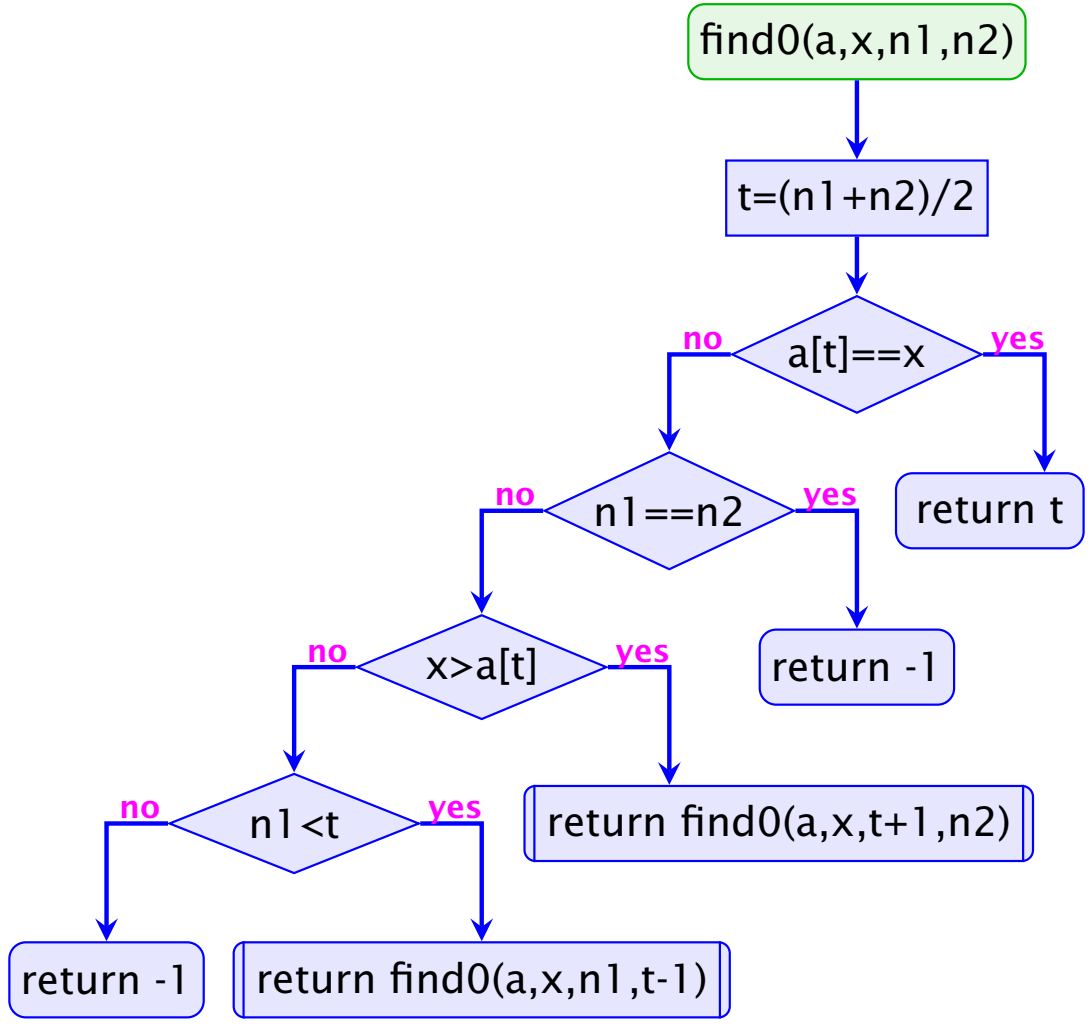

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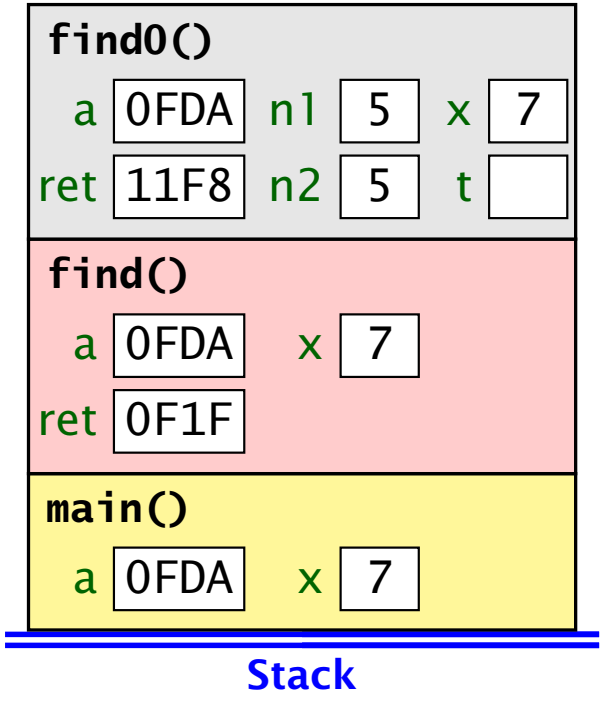

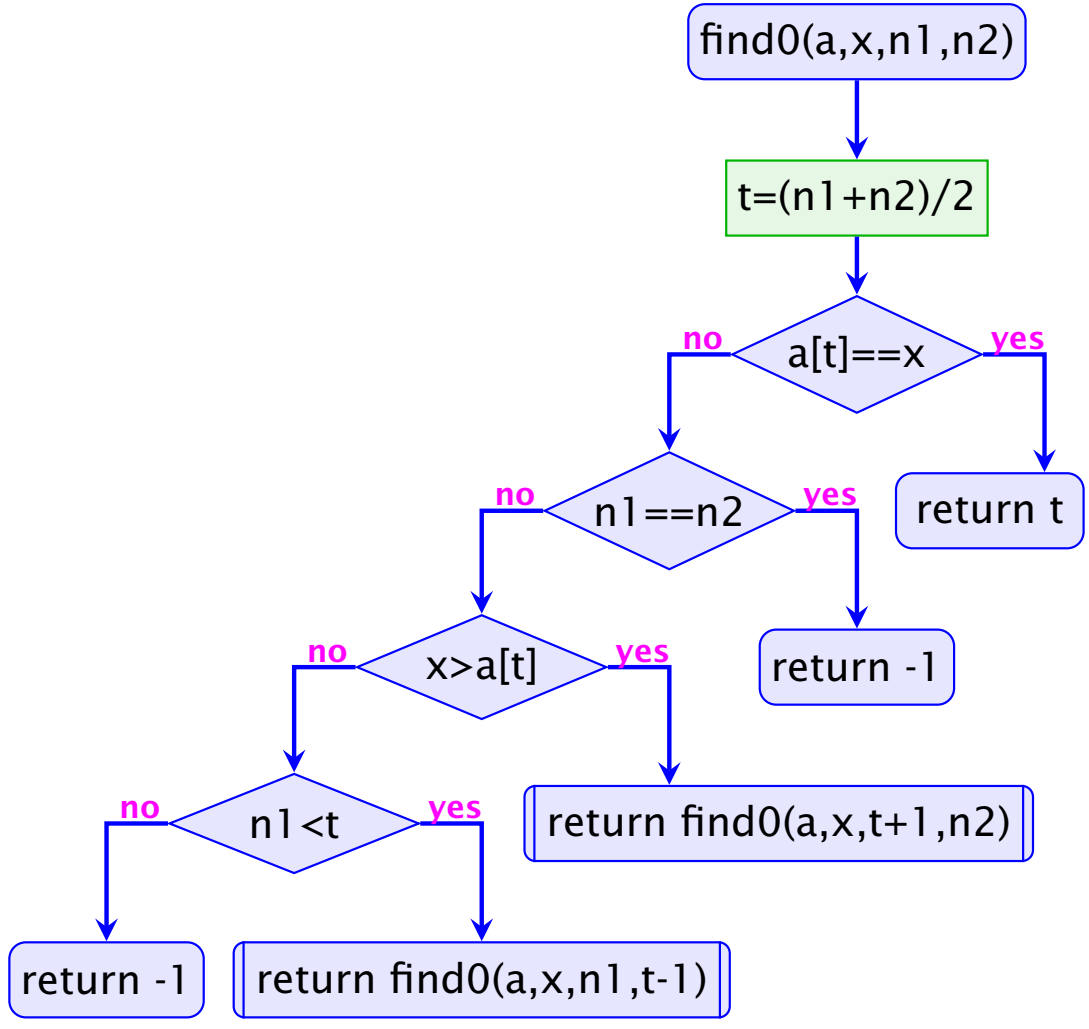

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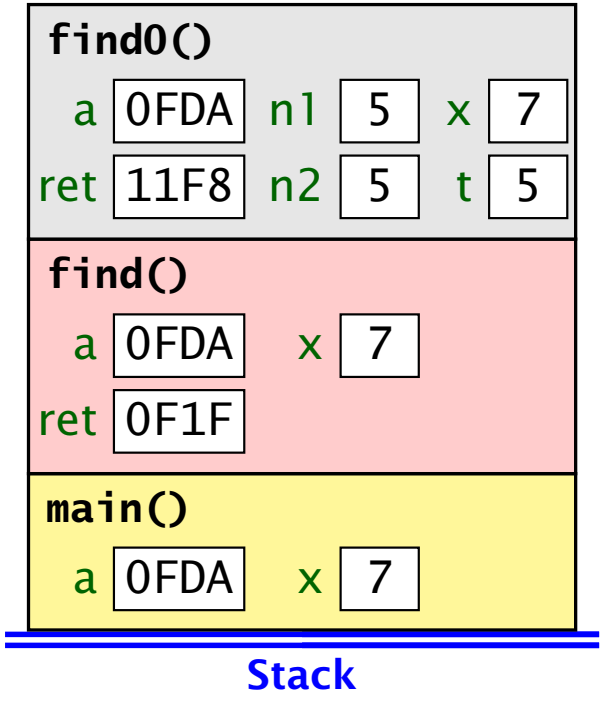

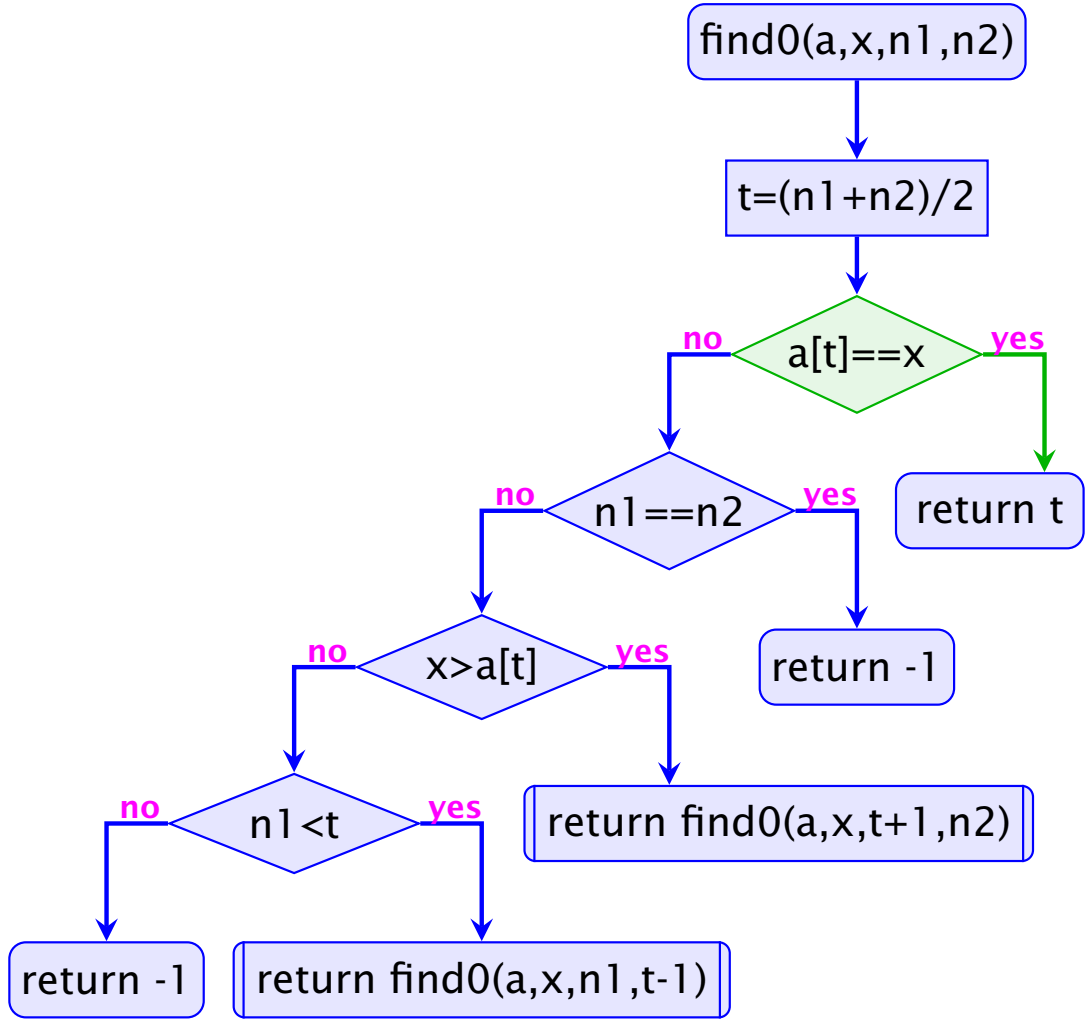

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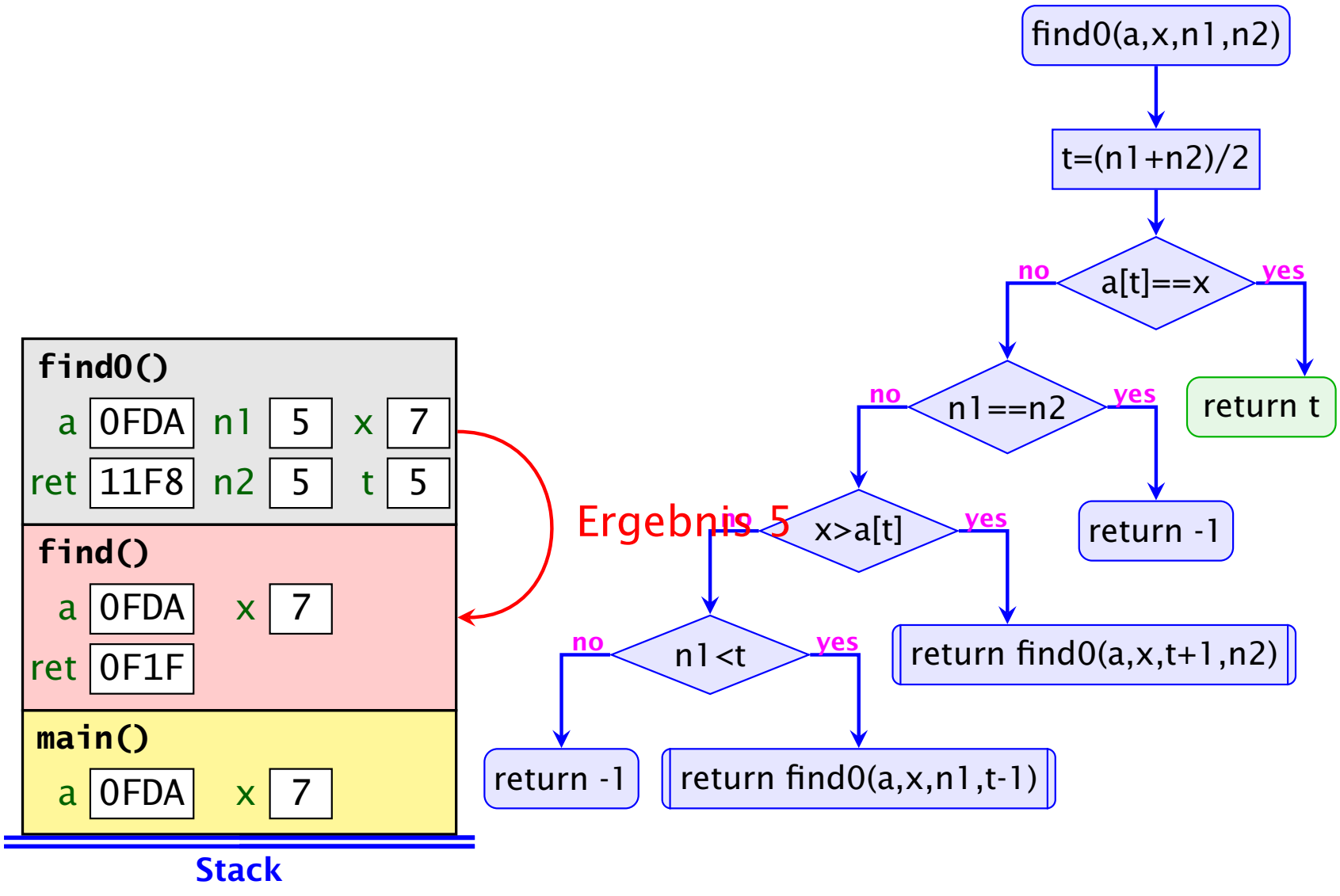

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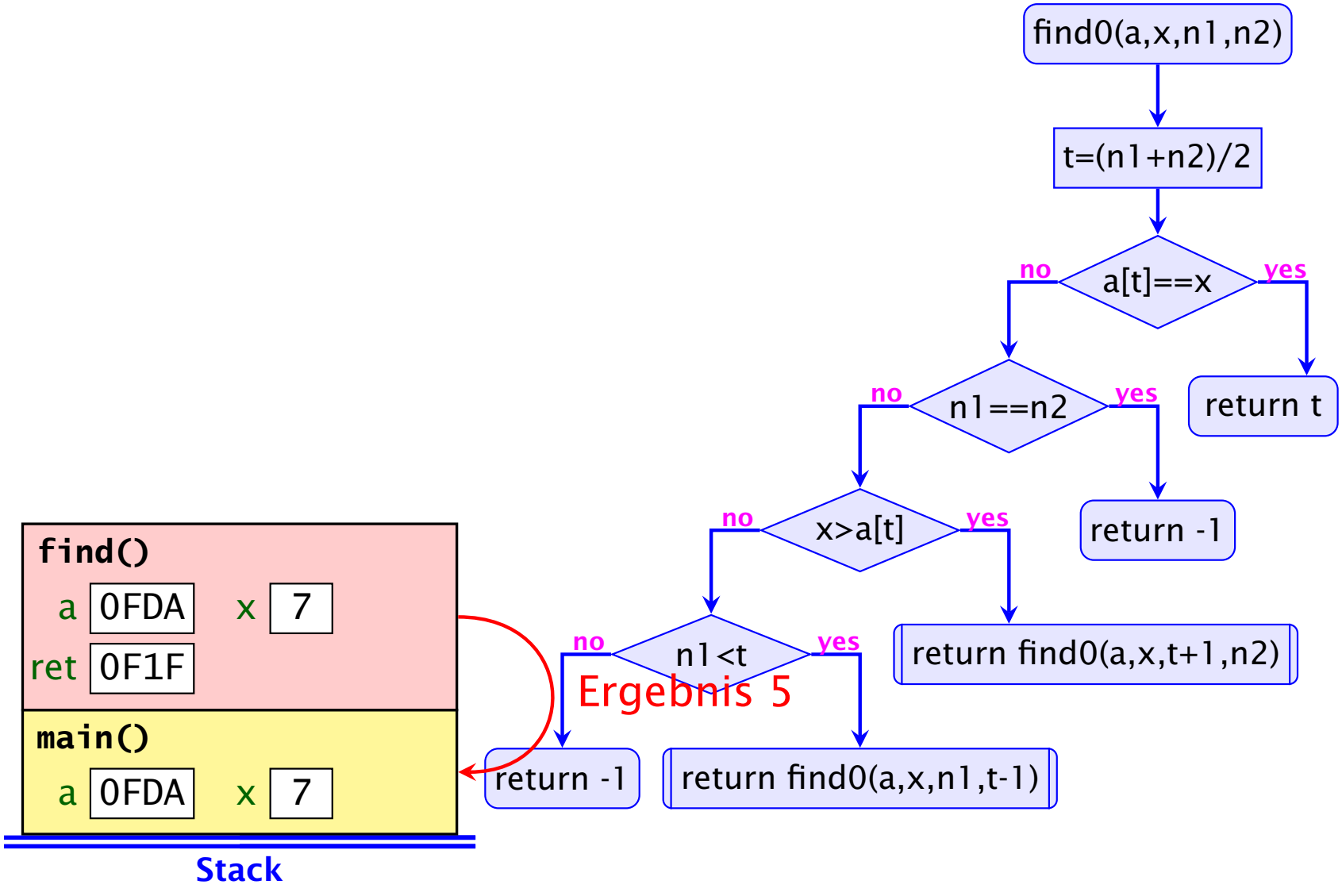
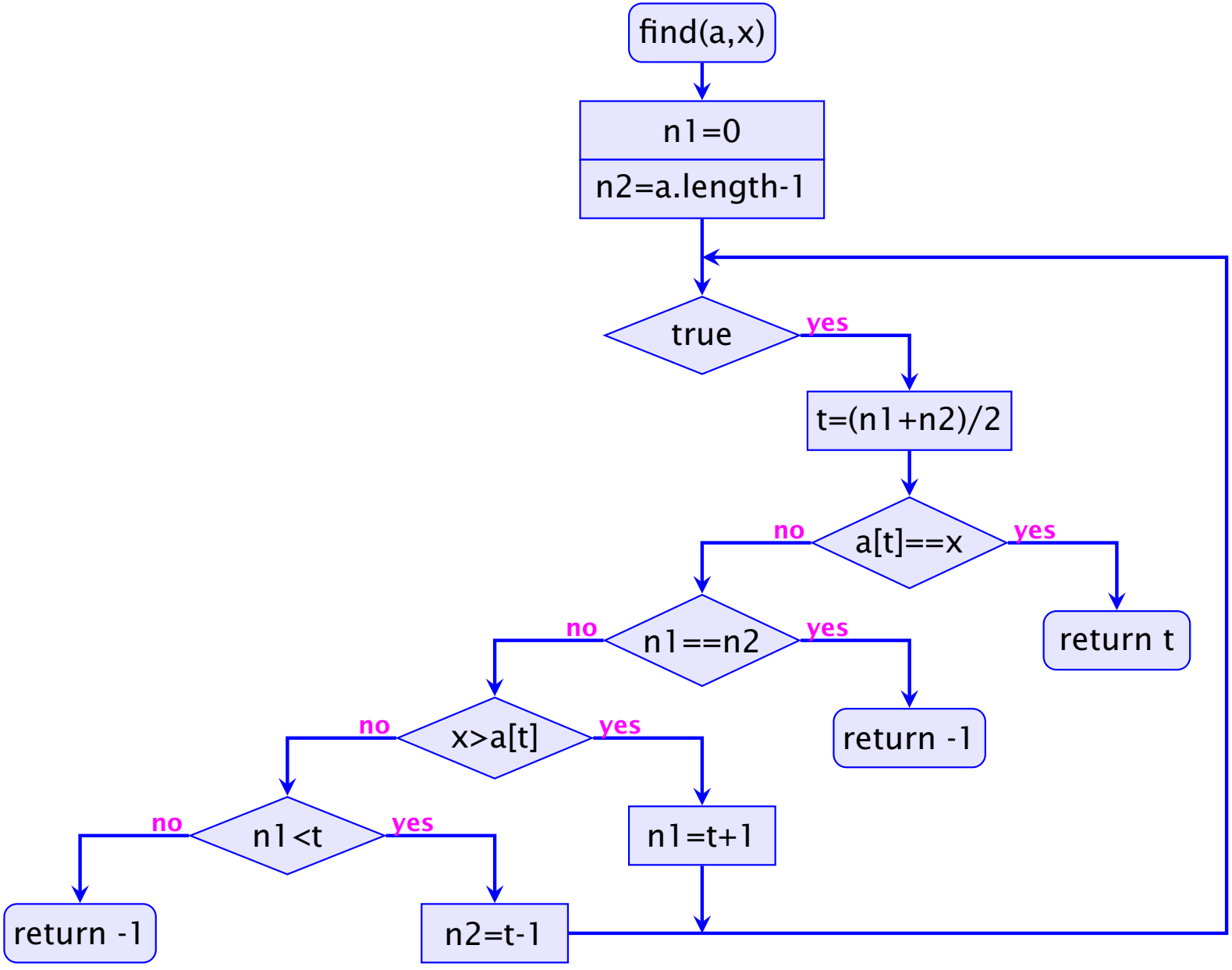


Endrekursion

Endrekursion kann durch Iteration ersetzt werden...

```
1  public static int find(int[] a, int x) {
2      int n1 = 0;
3      int n2 = a.length-1;
4      while (true) {
5          int t = (n2 + n1) / 2;
6          if (x == a[t]) return t;
7          else if (n1 == n2) return -1;
8          else if (x > a[t]) n1 = t+1;
9          else if (n1 < t) n2 = t-1;
10         else return -1;
11     } // end of while
12 } // end of find
```

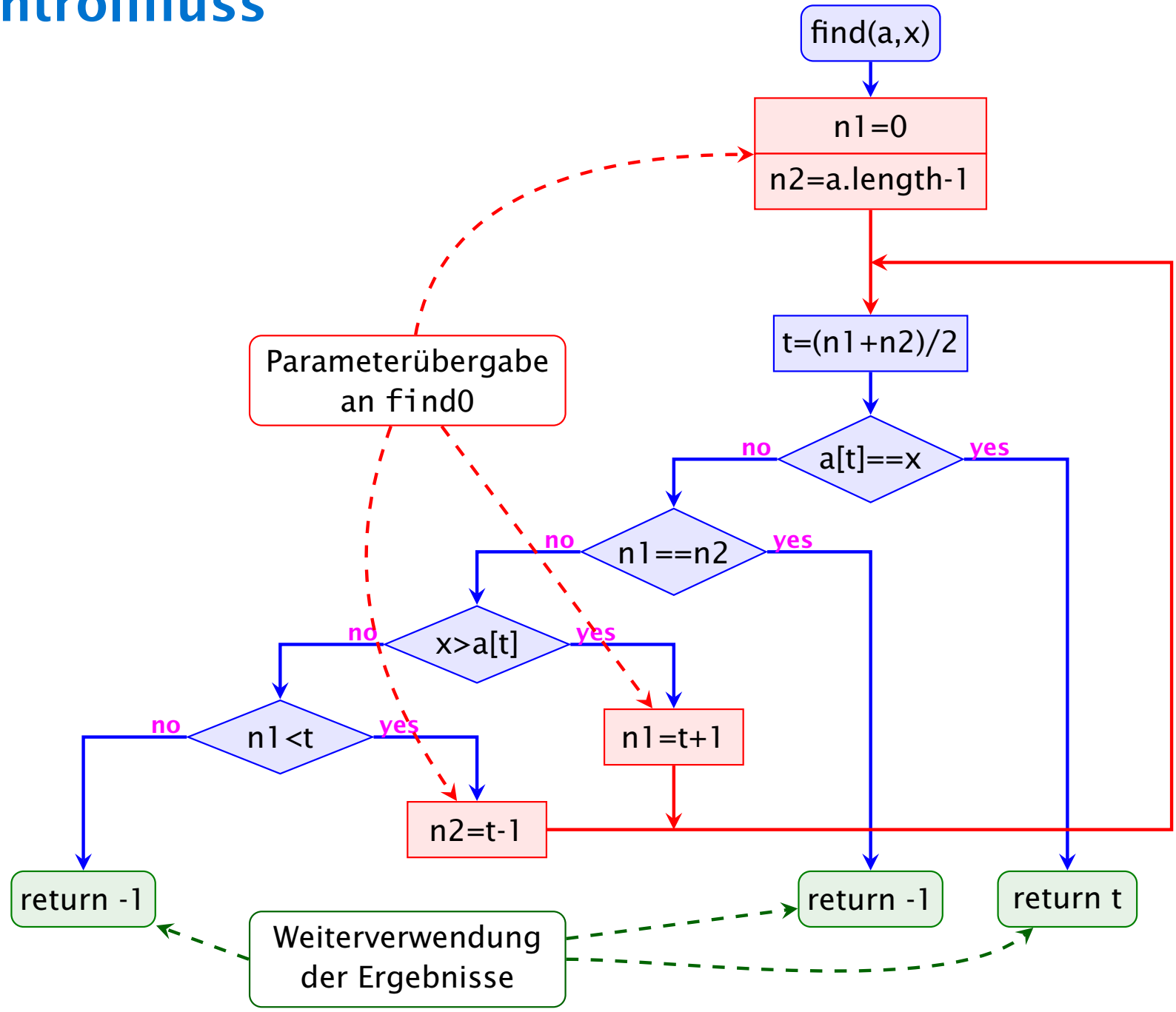
Kontrollfluss



Verlassen von Schleifen

- Die Schleife wird hier alleine durch die `return`-Anweisungen verlassen.
- Offenbar machen Schleifen mit mehreren Ausgängen Sinn.
- Um eine Schleife zu verlassen, ohne gleich ans Ende der Funktion zu springen, kann man das `break`-Statement benutzen.
- Der Aufruf der endrekursiven Funktion wird ersetzt durch:
  1. Code zur Parameter-Übergabe;
  2. einen Sprung an den Anfang des Rumpfs.

Kontrollfluss



Rekursion

Bemerkung

- Jede Rekursion läßt sich beseitigen, indem man den Aufruf-Keller **explizit** verwaltet.
- Nur im Falle von Endrekursion kann man auf den Keller verzichten.
- Rekursion ist trotzdem nützlich, weil rekursive Programme oft leichter zu verstehen sind als äquivalente Programme ohne Rekursion...