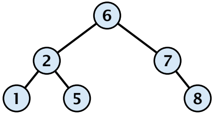
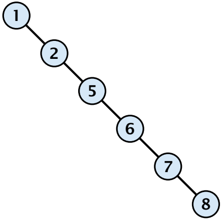
7.1 Binary Search Trees

An (internal) binary search tree stores the elements in a binary tree. Each tree-node corresponds to an element. All elements in the left sub-tree of a node v have a smaller key-value than $\mathrm{key}[v]$ and elements in the right sub-tree have a larger-key value. We assume that all key-values are different.

(External Search Trees store objects only at leaf-vertices)

Examples:



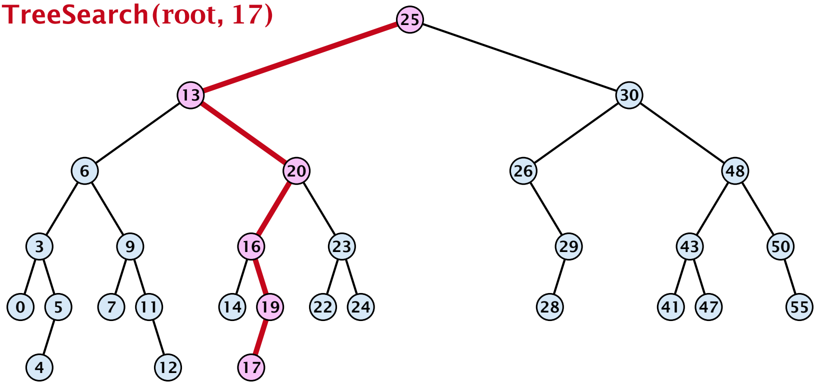


7.1 Binary Search Trees

We consider the following operations on binary search trees. Note that this is a super-set of the dictionary-operations.

- $T.\,\text{insert}(x)$
- $T.\,\text{delete}(x)$
- $T.\,\text{search}(k)$
- $T.\,\text{successor}(x)$
- $T.\,\text{predecessor}(x)$
- $T.\,\text{minimum}()$
- $T.\,\text{maximum}()$

Binary Search Trees: Searching



Algorithm 5 TreeSearch(x, k)

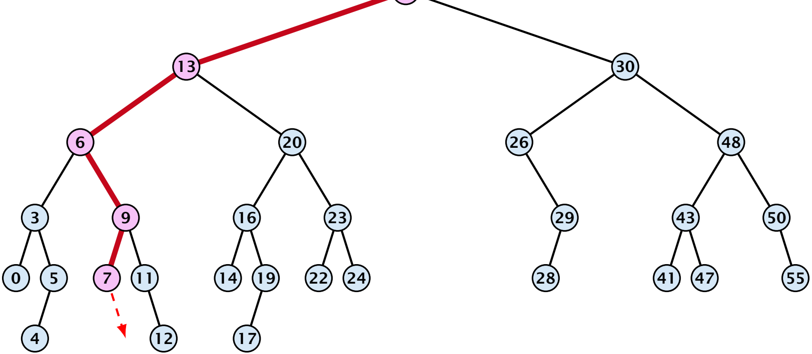
1: **if** x = null **or** $k = \text{key}[x]$ **return** x

2: **if** $k < \text{key}[x]$ **return** TreeSearch$(\text{left}[x], k)$

3: **else return** TreeSearch$(\text{right}[x], k)$

Binary Search Trees: Searching

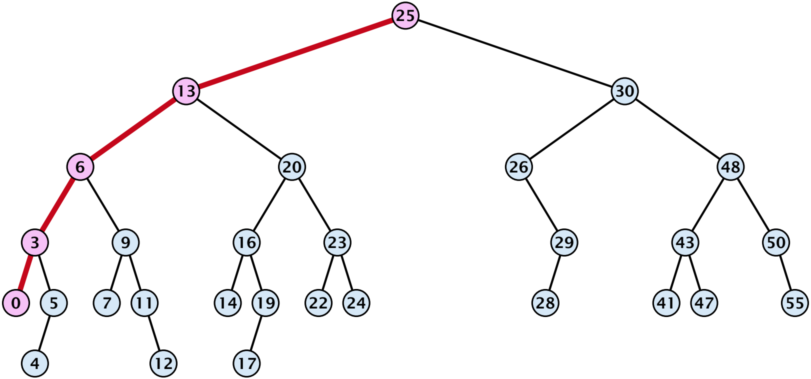
TreeSearch(root, 8)



Algorithm 5 TreeSearch(x, k)

if x = null **or** $k = \text{key}[x]$ **return** x
if $k < \text{key}[x]$ **return** TreeSearch$(\text{left}[x], k)$
else return TreeSearch$(\text{right}[x], k)$

Binary Search Trees: Minimum

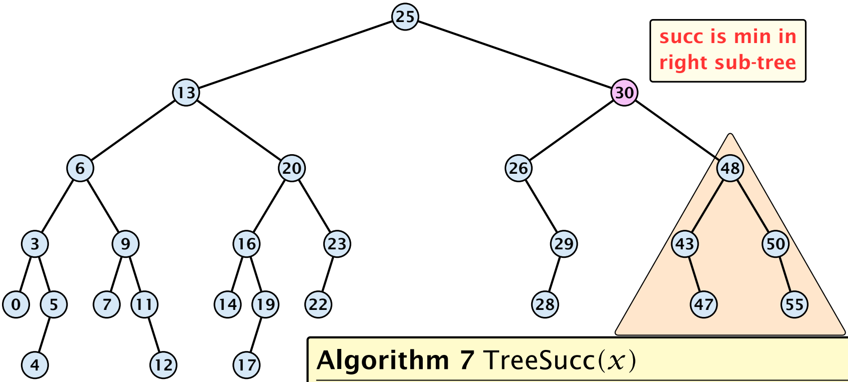


Algorithm 6 TreeMin(x)

1: **if** x = null **or** left[x] = null **return** x

2: **return** TreeMin(left[x])

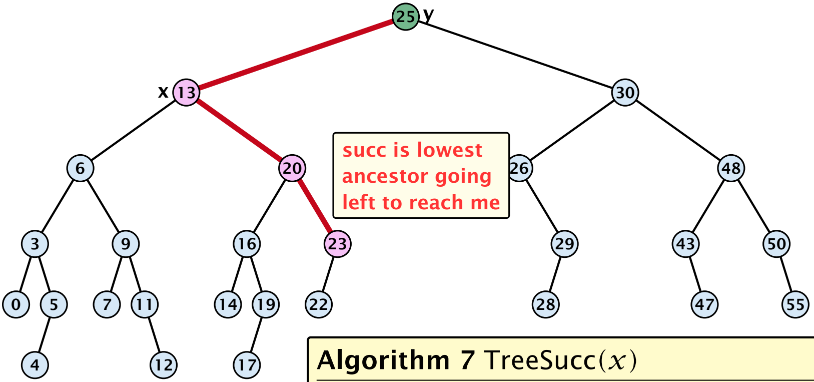
Binary Search Trees: Successor



Algorithm 7 TreeSucc(x)

if right$[x] \neq$ null **return** TreeMin(right$[x]$)
$y \leftarrow$ parent$[x]$
while $y \neq$ null **and** $x =$ right$[y]$ **do**
  $x \leftarrow y; y \leftarrow$ parent$[x]$
return y;

Binary Search Trees: Successor



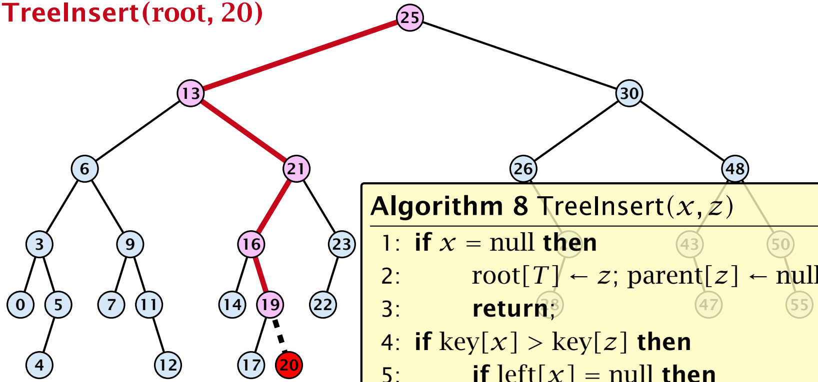
Algorithm 7 TreeSucc(x)

if right[x] $\neq$ null **return** TreeMin(right[x])
$y \leftarrow$ parent[x]
while $y \neq$ null **and** $x =$ right[y] **do**
    $x \leftarrow y$; $y \leftarrow$ parent[x]
return y;

Binary Search Trees: Insert

Insert element **not** in the tree.

TreeInsert(root, 20)

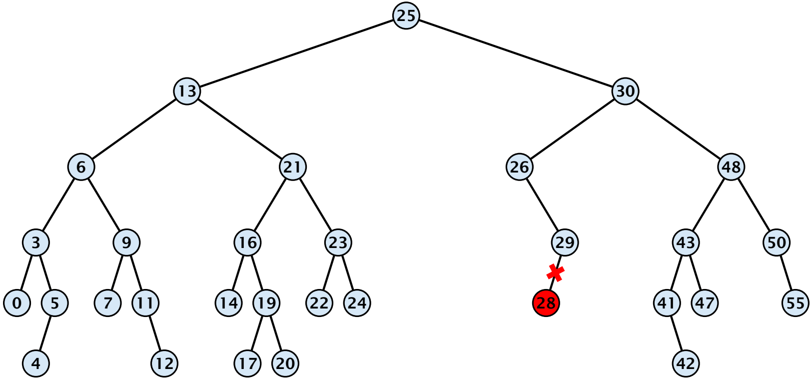


Search for z. At some point the search stops at a null-pointer. This is the place to insert z.

Algorithm 8 TreeInsert(x, z)

if $x = \text{null}$ **then**
    $\text{root}[T] \leftarrow z$; $\text{parent}[z] \leftarrow \text{null}$;
    return;
if $\text{key}[x] > \text{key}[z]$ **then**
    if $\text{left}[x] = \text{null}$ **then**
        $\text{left}[x] \leftarrow z$; $\text{parent}[z] \leftarrow x$;
    else TreeInsert$(\text{left}[x], z)$;
else
    if $\text{right}[x] = \text{null}$ **then**
        $\text{right}[x] \leftarrow z$; $\text{parent}[z] \leftarrow x$;
    else TreeInsert$(\text{right}[x], z)$;

Binary Search Trees: Delete

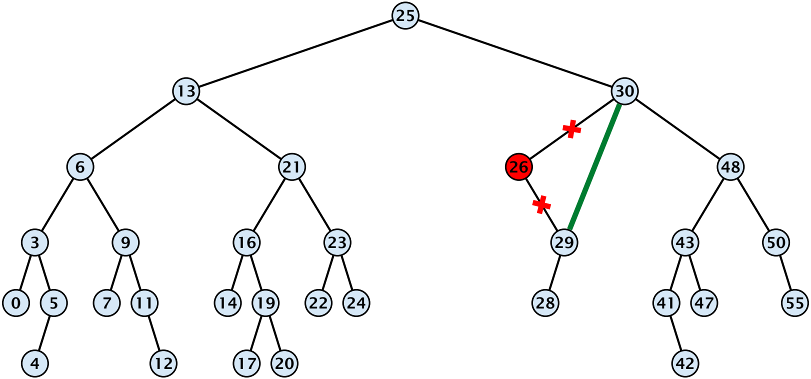


Case 1:

Element does not have any children

- Simply go to the parent and set the corresponding pointer to null.

Binary Search Trees: Delete

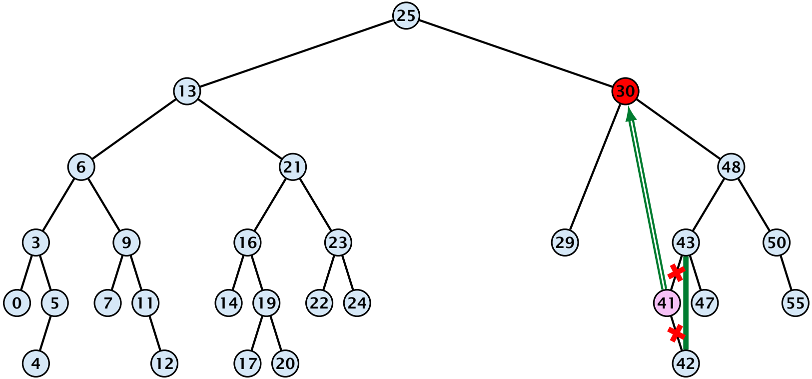


Case 2:

Element has exactly one child

- Splice the element out of the tree by connecting its parent to its successor.

Binary Search Trees: Delete



Case 3:

Element has two children

- Find the successor of the element
- Splice successor out of the tree
- Replace content of element by content of successor

Binary Search Trees: Delete

Algorithm 9 TreeDelete(z)

if left[z] = null **or** right[z] = null
    then $y \leftarrow z$ **else** $y \leftarrow$ TreeSucc(z); — select y to splice out
if left[y] $\neq$ null
    then $x \leftarrow$ left[y] **else** $x \leftarrow$ right[y]; — x is child of y (or null)
if $x \neq$ null **then** parent[x] $\leftarrow$ parent[y]; — parent[x] is correct
if parent[y] = null **then**
    root[T] $\leftarrow x$
else
    if y = left[parent[y]] **then** — fix pointer to x
        left[parent[y]] $\leftarrow x$
    else
        right[parent[y]] $\leftarrow x$
if $y \neq z$ **then** copy y-data to z

Balanced Binary Search Trees

All operations on a binary search tree can be performed in time $\mathcal{O}(h)$, where h denotes the height of the tree.

However the height of the tree may become as large as $\Theta(n)$.

Balanced Binary Search Trees
With each insert- and delete-operation perform local adjustments to guarantee a height of $\mathcal{O}(\log n)$.

AVL-trees, Red-black trees, Scapegoat trees, 2-3 trees, B-trees, AA trees, Treaps

similar: SPLAY trees.

Binary Search Trees (BSTs)

Bibliography

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[CLRS90] Thomas H. Cormen, Charles E. Leiserson, Ron L. Rivest, Clifford Stein:
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MIT Press and McGraw-Hill, 2009

Binary search trees can be found in every standard text book. For example Chapter 7.1 in [MS08] and Chapter 12 in [CLRS90].