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Randomized Algorithms

Exercise Sheet 9

Due: December 21, 2015
at 10:15, in class

Exercise 9.1 (10 points)
Consider a different version of the stable marriage problem in which men and women might be indifferent between certain options. There are two sets M and W with n men and n women, respectively. Each person has a preference list of the members of the opposite sex but there might be ties. For example, a woman w might prefer man m to man m' or she might be indifferent between them. Each preference list is organized in non-increasing order of desirability. Now, there are two different kinds of instabilities that might occur in a marriage.

Strong Instability: There exists a man m and a woman w such that they both prefer each other to their actual partners.

Weak Instability: There exists a man m and a woman w married with a woman w' and a man m', respectively, such that one of the following holds:

- m prefers w to w' and w prefers m to m' or she is indifferent between them, or
- w prefers m to m' and m prefers w to w' or he is indifferent between them.

Can we always find a marriage without strong instabilities? Can we always find a marriage without weak instabilities?

Exercise 9.2 (10 points)
Consider flipping a fair coin 100 times. Bound the probability of obtaining 55 or more heads. Use Markov's inequality, Chebyshev's inequality and a Chernoff bound.

Do the same for the probability of obtaining 60 or more heads.

Exercise 9.3 (10 points)
Consider a collection of n random variables X_i drawn independently from the geometric distribution with mean 2, that is, X_i is the number of flips of an unbiased coin up to and including the first occurrence of heads. Let $X = \sum_{i=1}^{n} X_i$.

Use a Chernoff bound to bound the probability that $X > (1 + \epsilon)2n$ for any fixed ϵ. To do this, a variable should be defined that can be expressed as the sum of independent Poisson (i.e., indicator) variables.

Exercise 9.4 (10 points)
Prove the following symmetric variation of the Chernoff bound for the lower tail. Let $X_1, X_2, \ldots, X_n$ be independent Poisson trials such that for $i = 1, \ldots, n$, $P[X_i = 1] = p_i$, where $0 < p_i < 1$. Then, for $X = \sum_{i=1}^n X_i$, $\mu = E[X] = \sum_{i=1}^n p_i$, and $0 < \delta \leq 1$,

$$P[X \leq (1-\delta)\mu] \leq \left(\frac{e^{\delta}}{(1+\delta)^{(1+\delta)}} \right)^{\mu}.$$