Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen (LEA)
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Spring Semester 2014
Problem set 8
May 28, 2014

Online and approximation algorithms

Due June 04, 2014 before class!

Exercise 1 (Spiral - 10 points)

Image yourself standing on a 2-dimensional grid searching for a point $t = (t_v, t_h)$ where t_v and t_h are vertical and horizontal distance from your current position. You do know that t is finite, however you do not know the exact values for t_v and t_h. Develop an algorithm for reaching t and prove its competitiveness.

Exercise 2 (Matching Game - 10 points)

Consider the following 2-person game: The players are given an arbitrary graph $G = (V, E)$ as well as a perfect matching M on G and take alternating turns. In each turn the player picks an edge $e \in E$ that was not picked before such that the selected edges form a simple path. The first player that is unable to choose such an edge looses the game.
Prove that there exists a winning strategy for the starting player.

Exercise 3 (Ranking - 10 points)

Let $G = (U \cup V, E)$ be a bipartite graph. Prove that the *Ranking* algorithm known from the lecture fulfills the following property: When fixing a permutation π on U the following methods produce the same matching:

- Method 1: Nodes of V arrive online and each node $v \in V$ is matched to an adjacent node $u \in U$ that has the lowest rank according to π.
- Method 2: Nodes in $V = \{v_1, ..., v_{|V|}\}$ are known in advance and nodes in U arrive in an online fashion according to π. Every node $u \in U$ is matched to an adjacent node $v \in V$ with the lowest index number.

Exercise 4 (Random - 10 points)

The online matching algorithm *Random* for bipartite graphs $G = (U \cup V, E)$ pairs each incoming node $v \in V$ with a node $u \in U$ that is chosen uniformly at random from the set of all neighbors of v. Show that the competitive ratio of *Random* is at most $\frac{1}{2}$.

Hint: Consider the family of graphs G_n with the following adjacency matrix A_n: For $1 \leq i, j \leq \frac{n}{2}$ or $i + j = n + 1$ $A_{i,j} = 1$, else $A_{i,j} = 0$. Nodes in U are represented by rows of A_n, nodes in V by columns of A_n.