Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen (LEA)
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Spring Semester 2014
Problem set 5
May 07, 2014

Online and approximation algorithms

Due May 14, 2014 before class!

Exercise 1 (LRU with potential function - 10 points)

Use the potential function technique to prove that LRU (the online paging algorithm that evicts the page that has been used least recently) is k-competitive.

Exercise 2 (RMTF$_p$ - 10 points)

In the lecture we saw the randomized online list update algorithm RMTF, that moves the requested element to the front of the list with probability $\frac{1}{2}$. We consider a generalized version RMTF$_p$ that moves a requested element to the front of the list with probability $p \in (0, 1)$.
Show that the competitive ratio of RMTF$_p$ is lower bounded at $\frac{1}{p} - \epsilon$ for any constant ϵ.

Exercise 3 (Modified BIT - 10 points)

Recall that the BIT-algorithm assigns a random bit to every item in the list before any request is served. When an item is requested its bit is flipped. If the flipped bit is 1 the requested item is moved to the front of the list, else its position does not change.
Consider the following modification: If the requested item is already in front of the list, we do not flip its bit.
Show that the modified algorithm is no longer $\frac{7}{4}$-competitive.

Exercise 4 (TIMESTAMP(0) - 10 points)

In the lecture the algorithm TIMESTAMP(p) for the list update problem was introduced. Upon a request to item x, it moves x to the front of the list with probability p or inserts x in front of the first item in the list that has been referenced at most once since the last request to x with probability $1 - p$.
Show that when $p = 0$ a requested item x never passes items that were requested 2 or more times since the last request to x.