Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen (LEA)
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Spring Semester 2014
Problem set 1
April 9, 2014

Online and approximation algorithms

Due April 16, 2014 before class!

Exercise 1 (Growing the cache - 10 points)

Recall that FIFO is the online paging algorithm that evicts the page that has been in the cache for the longest time, LRU is the online paging algorithm that evicts the page that has been used least recently and OPT is the optimal offline algorithm.
Consider the following request sequence

$$\sigma = \text{ABCDABEABCDE} .$$

Assume that we start with an empty cache. Calculate the cache during executions of OPT, LRU and FIFO on σ for cache sizes 3 and 4. Count the number of page faults during the executions. What do you notice?

Exercise 2 (Ski Rental - 10 points)

The ski rental problem is defined as follows: Assume that renting a pair of skis costs 1 per day, buying a pair of skis costs b. Every day we have to decide, in an online fashion whether we want to continue renting skis for another day or buy a pair of skis. At some unknown time D, we will break our leg and have to quit skiing. Our goal is to minimize the cost of skiing.

(a) Develop a strongly $(2 - \frac{1}{b})$-competitive online algorithm ALG for the ski rental problem and prove its competitiveness. (a strongly c-competitive algorithm ALG satisfies $\text{ALG}(\sigma) \leq c \cdot \text{OPT}(\sigma)$ for all request sequences σ)

(b) Show that there is no deterministic online algorithm with a competitive ratio better than $(2 - \frac{1}{b})$.

Exercise 3 (First-in First-out - 10 points)

Recall that FIFO is the online paging algorithm that evicts the page that has been in the cache for the longest time.
Prove that FIFO is k-competitive.